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Don't be a Square: The processing mechanisms characterising the elemental dimensions of width and height

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Abstract

What are the geometric and information processing characteristics of elementary figures composed of simple physical dimensions? There have been a number of investigations of perception of rectangles, including debate about configurality (e.g., integrality and gestalt properties) as well as the prime perceptual dimensions. Yet, because of ambiguity even in the "right" definition of configurality and an absence of penetrating methodologies, there is still little known concerning the information processing of these patterns. To this end, the present study brings together two separate theory-driven methodologies, general recognition theory (GRT) and systems factorial technology (SFT). The first attacks the problem of dimensional interactions while the latter seeks to uncover process characteristics such as architecture, decisional stopping rules, and workload capacity. The same observers and as much as possible, the same stimuli were used in both approaches. Through our GRT analyses, we found strong evidence for dependencies between the percepts of height and width on both within-stimulus and cross-stimulus bases. Height perception was better with narrow widths and width perception was superior with short heights. In addition, a significant positive within-trial correlation of dimensions was evidenced within squares but not with rectangles. Our SFT initiative uncovered consistent signatures of parallelism paired with super capacity, the latter appearing both through the traditional conditioning on being correct and still present when modest speed accuracy trade-off was accounted for. Thus, the SFT and GRT inferences were quite compatible with a plausible cause of the positive correlations being across-channel facilitatory interactions which led to super capacity processing.

Keywords

Configurality; rectangle perception; integrality; architecture; capacity; perceptual separability; perceptual independence; decision bias; perceptual sensitivity

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The ability of the human mind to weld what are distinct aspects of dimensions of the physical world into more unified constructs has been of interest in scientific psychology for at least 100 years and reaches back considerably further within the realm of philosophy (e.g., see Boring, 1950). The modern scientific approach is called *gestalt psychology*. Ideas and experiments exploring holistic phenomena extend from natural patterns of high complexity like faces, to combinations of rather elemental features. Configural aspects of perception of even simple combinations of simple features have been put forth. One of these is perception of parallelepipeds, especially rectangles and squares. The present work focuses on the latter class of forms and aims to probe the geometric and information processing characteristics of elementary figures composed of simple physical dimensions.

We begin by briefly reviewing some of the pertinent history of the field of configural perception and some remaining issues of our concerns. Then, we outline the particular approach we take for the current study after a brief introduction of the two major methodologies of our interests and a review of the background regarding con-

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figurality of rectangles. Readers who wish to skip the historical material may jump to the next section.

Brief history of configural perception

The original gestalt approach invented a number of convincing perceptual experiences of holistic qualities of objects that went beyond what would be expected from independent processing of the constituent dimensions or features. However, as Boring (1950) observes, these could in large part, be interpreted as *experimenta demonstranda* rather than instruments of hypothesis testing or even open-ended explorations. This situation has begun to alter dramatically in recent years with publications such as Wagemans' compendius *The Oxford Handbook of Perceptual Organization* (Wagemans, 2015). Please see Townsend and Wenger's (2015) chapter in that volume which presents in broader scope our approach to gestalt phenomena.

In their place, and not to be easily dismissed, are *operational definitions*. With some unavoidable oversimplification, an operational definition specifies the occurrence or evidence of a concept by an experimental result. A major figure who championed operational definitions was Wendell R. Garner.[1] One of his important trains of research intersected with the aims of gestalt psychology is what he called *perceptual integrality*. Perceptual integrality, informally defined, refers to an absence of perceptual independence of two or more psychological dimensions. Somehow, two or more dimensions are fused into something that goes beyond the simple independent set of dimensions.

Scores of studies over the years have been dedicated to empirically testing perceptual integrality using various sets of physical dimensions and experimental contexts (i.e., Amishav & Kimchi, 2010; Ganel & Goshen-Gottstein, 2004; Garner, 1974). Much of the work employed several operational definitions of perceptual integrality (see the compendious review by Algom & Fitousi, 2016) and then experimented to see whether they converge in their implications. Such strategies have led to better comprehension of configural processing (e.g., Von Der Heide et al., 2018) and will continue to do so. However they suffer, like all strategies, from shortcomings. One limitation of operational definitions is that distinct operational definitions typically employ different experimental designs and perceptual tasks and thus open up the possibility that different forms of configurality and processing mechanisms might be called upon.

Another prime and intuitive avenue of investigation has been that of multidimensional scaling (MDS), which endeavours to use observables like similarity judgements to embed the perceived stimulus patterns (e.g., Kruskal, 1978; Shepard, 1980; Torgerson, 1952) into a multidimensional space usually assuming some kind of metric. More rarely, other observables like confusion frequencies or response times, can be employed as input (Nosofsky, 1986; Podgorny & Garner, 1979; Townsend, 1971).[2] Although MDS concepts and tools will undoubtedly continue to play a valuable role in coming to an understanding of geometric and topological characteristics of human perception and cognition, there are shortcomings here too.

One limitation is that almost all MDS approaches are deterministic, that is, devoid of the probability which forms an inherent aspect of human thought and behaviour. Another is that when using subjective judgements, for instance, of similarity or dissimilarity, it seems almost certain that the resulting scales must be treated as ordinal. This is because the steps necessary to verify, for instance, the statutes of an interval scale have rarely been followed in conjunction with multidimensional scaling (e.g., Krantz et al., 1971). This results in ongoing debates of whether and to what extent, ordinal (i.e., monotonic) transformation can and should be employed before the programmes which attempt to place stimulus-related points into a multidimensional space (e.g., Schönemann et al., 1985). Last but not least, there is an absence of critical psychophysical or psychological information processing mechanisms that presumably underpin the establishment of these spaces.

In recent years, perceptual integrality and other gestalt-related concepts have received more attempts that are mathematical but aim at critical issues of the information processing attendant to configural perception. One vein of research into these issues has focused on its underlying principles, engaging what the innovators have referred to as theory-driven methods. Such works try to uncover the underlying cognitive principles of multidimensional processing from a certain type of strong-scale, behavioural measurement, using either response frequencies (e.g., Ashby & Perrin, 1988; Ashby & Townsend, 1986) or response times (e.g., Schweickert & Dzhafarov, 2000; Townsend & Nozawa, 1995). Furthermore, the latest works following this approach have been dedicated to expanding the theoretical methods that account for both aforementioned types of behavioural measurement (e.g., Ashby, 2000; Eidels et al., 2015; Fific et al., 2010; Silbert & Houpt, 2014b; Townsend & Altieri, 2012; Townsend et al., 2012).

Systems identification in perception, cognition, and action: general recognition theory and systems factorial technology

Lying within this general line of attack, the current study seeks to provide a deeper insight into the underlying cognitive principles of how integrality occurs. The theory-driven methodologies brought to bear here can assist in answering this query. Two separate theory-driven methodologies, namely the general recognition theory (GRT,

Table 1. General meaning of the GRT terms.

Type of independence	Term	Meaning
Processing independence	Perceptual separability	The perception of each dimension does not change with the information of other dimensions.
	Perceptual independence	The perception of each dimension is independent in the presence of a particular stimulus.
	Decisional separability	The decisional criterion of each dimension does not change with the information of other dimensions.
Observable response independence	Marginal response invariance	The response behaviour of each dimension does not change with the values of other dimensions.
	Report independence	The response behaviour of each dimension is independent in the presence of a particular stimulus.

Ashby & Townsend, 1986; Ashby, 1992) and the systems factorial technology (SFT, Little et al., 2017; Townsend & Nozawa, 1995), are brought together to provide global insights into how people perceive the elementary figures. To focus the scope of the paper, we provide a brief introduction of these two methodologies here, and include more detailed tutorials for readers unfamiliar with them in Supplementary Appendix A.

GRT was originally invented as an extension of signal detection theory (SDT; Green & Swets, 1966) to include multidimensional perception and thereby provides a potential bridge from the geometry to the perceptual processes and does so in a stochastic manner. It provides a method to assess various types of processing independence (i.e., perceptual separability, perceptual independence, decisional separability) using tests of response independence (i.e., marginal response invariance, report independence). Table 1 summarises the general meaning of these independence terms and conditions. The formal definitions of these terms specified for static and timed dynamic systems are found in Supplementary Appendix A.

The initial approach, and still an important facet of the methodology, was a static (time not included) theory-driven methodology for assessing various types of independence including perceptual separability (see Definition 1 in Supplementary Appendix A), perceptual independence (Definition 2 in Supplementary Appendix A) and decisional separability (see Definition 3 in Supplementary Appendix A) in multidimensional cognitive processing with response-frequency-based measures including marginal response invariance (Definition 4 in Supplementary Appendix A) and report independence (Definition 6 in Supplementary Appendix A). Figure 1a summarises the static GRT implications of these various independence from empirical measures. Certain types of perceptual dependencies associated with failures of the analogous independencies can and have been shown to be associated with specific kinds of gestalt or integral phenomena (see e.g., Townsend & Wenger, 2015, for a review of the concepts and some examples).

The original GRT theory was derived without any distributional constraints, but in practical application it is often exploited within the class of multidimensional signal detection models (see Ashby, 2000). For instance, using multivariate Gaussian distributions, Kadlec and Townsend (1992) showed that if such a system satisfies perceptual separability, then marginal d's of a dimension should be equivalent across different levels of the other dimension. Subsequently, marginal d's have been employed as an essential empirical measure to assess perceptual separability along with other theoretical tools (i.e., Cornes et al., 2011; Kadlec & Hicks, 1998; Wenger & Rhoten, 2020).

More recently, time-dynamic versions of this theory have been put forth to account for both response frequencies and response times referred to by the acronym RTGRT (Ashby, 2000; Townsend et al., 2012). RTGRT provides reinforcement of the inferences and further insight into underlying mechanisms (see Figure 1b for dynamic GRT implications and Supplementary Appendix A for detailed definitions of terms). It does so within the frame of a class of *accrual-halting parallel models*, postulating a set of parallel channels. As a result, the RTGRT applications so far are drawn from the postulate of parallel processing (Townsend et al., 2012) but the foundations, as with the static version, are parameter and distribution free.

To this end, this study's other fundamental direction is the identification of the mental architecture and related characteristics responsible for multidimensional information processing (Townsend & Ashby, 1983). This body of tools includes identification of whether the perception of each dimension occurs in a serial, parallel, or more complex fashion, and whether the interactions between dimensions facilitate or impair the processing efficiency of each other. The general methodology most appropriate for such ventures is SFT.

SFT is a theory-driven methodology for assessing fundamental cognitive characteristics including mental architectures (serial vs. parallel), stopping rules (self-terminating vs. exhaustive), and workload capacity (limited, unlimited, vs. super) using response-time-based measures including

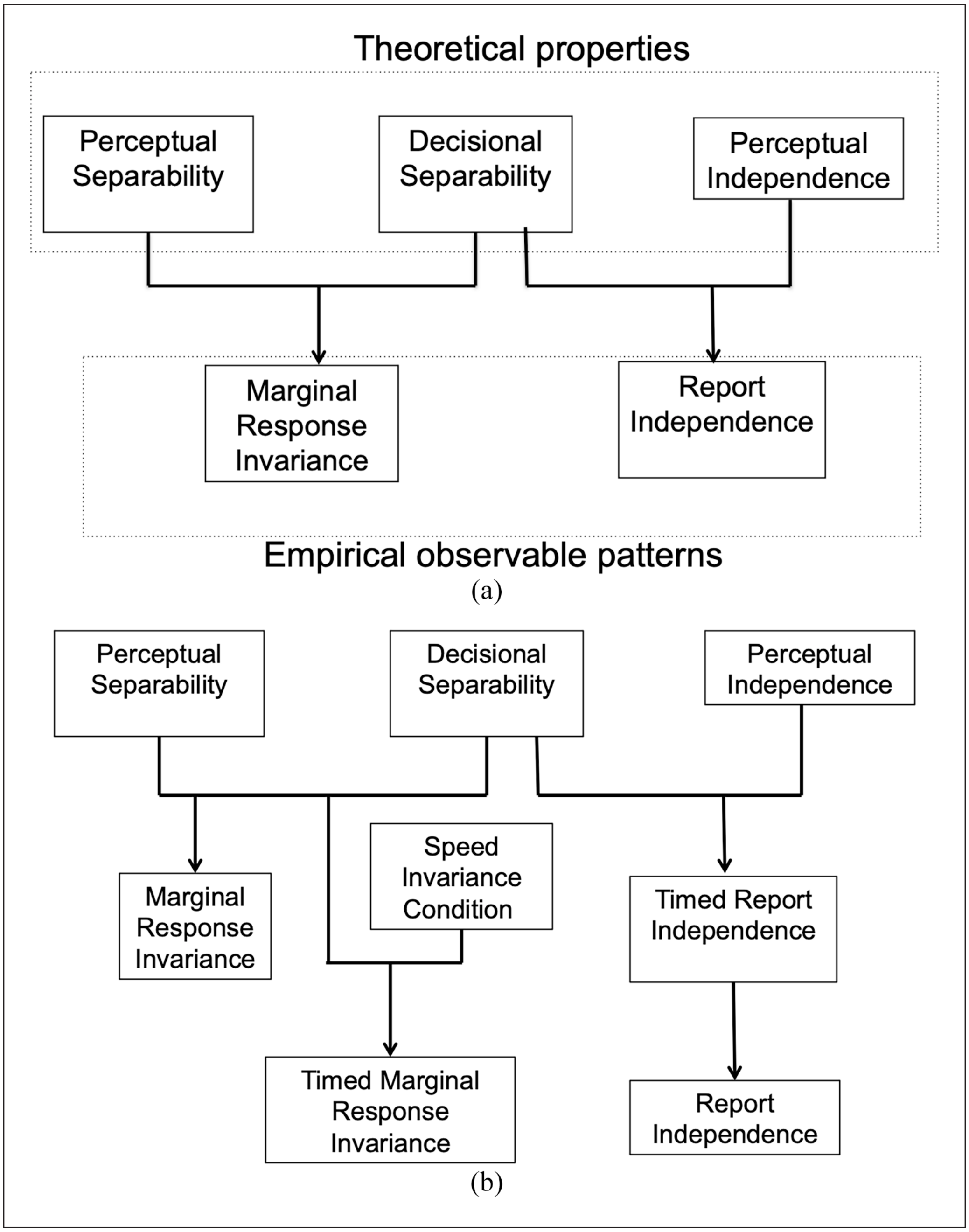

Figure 1. (a) Implications of the theoretical static-GRT constructs for the empirical observations. (b) Extended implications of the theoretical RT–GRT for the empirical observations.

the survivor interaction contrasts (see Definition 9 in Supplementary Appendix A) and the capacity coefficients (see Definition 10 in Supplementary Appendix A). Recent efforts have also been devoted to extending SFT to account for both response times and response frequencies (Fific et al., 2010; Townsend & Altieri, 2012).

However, the two separate toolboxes, GRT and SFT still assess completely distinct information processing mechanisms and properties. As intimated, a major thrust of our current theoretical enterprise is to combine GRT and SFT as an interlocking set of tools. The present effort is a step in that direction.

Configurality in rectangle perception

We selected the relatively elemental geometric forms of rectangles for our inquiry—simple but clearly involving more than one dimension. In many avenues of search for holism or integrality, there are straightforward predictions from prior researches. Simple though rectangles and squares may be, the guidance from preceding literature is scant.

One might think that rectangles' ready decomposition into length and width might afford a good possibility for not only the operable perceptual dimensions (e.g., through multidimensional scaling) but also dimensional separability and independence. However, a substantial portion of the studies supported some type of configurality between height and width, including studies using multidimensional scaling (e.g., Krantz & Tversky, 1975), Garnerian methods (Garner, 1974; Macmillan & Ornstein, 1998), and other methods such as modelling (e.g., Monahan & Lockhead, 1977). Yet, the interpretation of this concept was far from universal. As we learn more about what structures and information processing mechanisms actually relate to configurality, it is to be hoped that elementary dimensions like height and width engage less complex versions of those accompanying complex percepts such as faces.

The multidimensional scaling literature on rectangles (e.g., Krantz & Tversky, 1975) finds a debate over the identity of the true psychological dimensions. Most often the argument is between length and width versus area and shape. Beyond the usual numerical procedures, the foundational measurement sector has developed axioms for constructing certain types of metrics such as the power metrics which include the Euclidean metric and the important city-block metric (e.g., Krantz et al., 1971). Krantz and Tversky (1975), following up on work of Wender (1971), concluded that the pertinent dimensions were shape and area but that these interacted. Schöneman and colleagues (Lazarte & Schönemann, 1991; Schönemann et al., 1985; Schönemann & Lazarte, 1987), taking into account the findings of Borg and Leutner (1983), lodged a number of criticisms against the conclusions of Krantz and Tversky, and argued that the results are more parsimoniously interpreted as supporting width and height, with much less interaction.

Overall, the debate over width and height versus shape and area lasted over many years and never seems to have been settled. The general weight of opinion seems to favour width and height. Evidence for some kind of configurality varies from the importance of attention (Piaget, 1969) to subadditivity both interdimensionally and intradimensionality (Schönemann et al., 1985; Schönemann & Lazarte, 1987). For instance, Piaget (1969) found that his notions of attention correctly predicted that height would be overestimated if paired with a more narrow width than with a wider width.

One of the few relatively common inferences regarding holism was that area and shape were important psychological features and that shape dominated area. One possible extension of that concept is that even if we calibrate the stimuli so that, as individual patterns, length and width are equally perceptible, it could be when, say four stimulus patterns, two square and two rectangles, one tall and one wide are used, the perceptual sensitivity to the rectangles could be greater than that for squares.

Several past studies, taking a Garnerian tack, also inferred integrality mainly from the observations of redundancy gains in rectangle perceptions. For instance, Felfoldy's (1974) use of Garner's filtering method found interference while the correlated condition found some facilitation leading to an inference of integrality for rectangles. And, results from Dykes and Cooper (1978) came to a similar conclusion. However, we must keep in mind the fact that redundancy gains in the correlated-dimension tasks are predicted even by ordinary, independent parallel-channels models (e.g., Egeth, 1966; Raab, 1962; Townsend et al., 2020). One must show that the redundancy gains exceed the theoretical predictions on statistical facilitation (resulting in what is now known as super workload capacity) to infer holistic processing (Townsend & Nozawa, 1995; Townsend & Wenger, 2004).

Also, deficits because of failure to isolate attention to a single dimension in the filtering condition probably calls on distinct mechanisms (i.e., attentional interference) from facilitation with redundant signals in the studies using the conventional Garnerian method. In our opinion, the most germane of the past studies on rectangles to our present aims, was a report by Macmillan and Ornstein (1998) also implementing Garner's operational concepts. They avoided our criticism that distinct experimental operations could, a priori, involve very different underlying processing mechanisms or characteristics.

This was accomplished through GRT principles developed by Maddox (1992) and Ashby and Maddox (1994) wherein Garner's definitions are interpreted as involving logically diverse decision rules within the same or modestly perturbed multidimensional spaces. The usual response frequency data from GRT designs were employed but not the additional observables associated with response times (Ashby and Maddox, 1991, 1994; Maddox & Ashby, 1996). The parsimonious and informative outcome was a set of inferences within that single framework. From the current perspective, a regrettable omission was a complete identification paradigm that would have matched our GRT design. In addition, perceptual independence was assumed rather than tested so the positioning of the distributions in two-dimensional space formed the primary basis for inferences about integrality.

As observed above, there is almost no literature on the information processing characteristics of rectangle perception. From a rather extreme point of view, a truly unified holistic percept should admit no decomposition into separate dimensional contributions to a metric. For instance, Little et al. (2013) referred to the holistic process as a combination of perceptual information from each dimension even before the decision phase (i.e., coactivation). This purist criterion is not sustained by any of the scaling studies.

Some researchers have argued for a version of holism based on a so-called *blob model* wherein the observer perceives, early on, a rather vague and undetailed form of the stimulus. Subsequently, if conditions like contrast and temporal exposure are sufficient, finer grained detail becomes available (Lockhead, 1972; Monahan & Lockhead, 1977). The blob conception can be interpreted in terms of the more precise language of larger vs. smaller spatial frequencies (Ginsburg, 1984). There is some evidence for blob-like processing in other venues, such as letter perception (e.g., Lupker, 1979; Townsend et al., 1984, 1988) and short-term memory for faces (Wenger & Townsend, 2000).

It remains to be seen whether the blob conception can effectively produce the wealth of processing characteristics presently uncovered by GRT and SFT. Occasionally, researchers will seek to infer serial (e.g., Felfoldy, 1974) or parallel operations (e.g., Dykes & Cooper, 1978). But in light of the ubiquitous challenge of model mimicry in the absence of strong theory-driven methodologies as well as the contradictory conclusions, it is hard to settle their differences.

Current study

As mentioned above, the fields of gestalt psychology have always suffered from a general lack of quantification even for objects as simple as rectangles. Combining GRT and SFT as an interlocking set of tools, the current study aims to uncover the essential principles related to integrality of simple features.

We composed a set of rectangular stimuli and had observers complete an identification task and a classification task. In the identification task, we utilised the toolboxes of GRT and investigated whether the information processing of height and width interacted with each other. If so, was the interaction positive or negative and did the dependency occurred in the course of perception, decision, or both?

In the classification task, we utilised the toolboxes of SFT and investigated whether height and width are processed in parallel and the proper decisional stopping rule is engaged. In addition, with such elemental features, would presentation of them both together cause a drain in workload capacity? Or is it possible that the gestalt properties of the figures may lead to super capacity?

In addition, we employed different sets of payoff matrices to assess what role that decisional biases might play with regard to the perceptual integration of height and width, as well as to help mitigate the portent of model mimicry as uncovered by R. Thomas and Silbert (Silbert & Thomas, 2013, 2017; Thomas, 1996).[3] It should also be interesting to learn how manipulation of response bias affects response preference, a novel manipulation in the SFT literature.

A brief preview of our findings indicates configural features of rectangle perception as was revealed by various types of interactions, appeared in both sensory and decisional mechanisms in the GRT analyses. There was a bias in favour of rectangles over squares, higher accuracy on a dimension if the other dimension was lesser in magnitude, and a positive within-trial correlation with squares but not rectangles. In addition, SFT uncovered parallel rather than serial processing accompanied by the logically appropriate exhaustive stopping rule. Super capacity was revealed and this property is compatible with the GRT finding of the positive correlations of height and width found to some extent with both squares and rectangles but especially with squares. More detailed summaries can be found in section "General summary and discussion."

Datum collected in this study is available on the Open Science Framework at the link: https://osf.io/srpva/?view only=182027d819214e9f92598d64e62d8cdc.

Method

Participants

Eight observers[4] (three males and five females) from the Indiana University community were paid to participate in this study. Their ages ranged from 19 to 25 years and the average age was 21.5 years. All observers reported having normal or corrected-to-normal vision.

Observers were reimbursed a US$10 base rate in addition to a US$1–3 performance-based bonus for their participation in each session, as well as an extra US$20 bonus for the completion of all sessions. All observers participated in a total of 18 sessions, including 9 sessions of the unbiased condition and 9 of the biased condition.

Materials

A total of nine rectangular stimuli were used for each observer. Each rectangular stimulus was composed of a combination of magnitude levels in height and width dimensions. Each dimension had three levels (base, low, and high), which were individually calibrated for each observer at the beginning of each payoff condition (see section "Design" below) to prevent ceiling effects in addition to producing relatively similar perceptual sensitivities. Specifically, the low levels (−) in both dimensions were set to be the same for all observers. The high levels (+) were staircase calibrated with a complete identification session, which ensured that the marginal d's on both height and width were within the range of 0.75–1.5. Furthermore, the base levels (0) were staircase calibrated with a classification session to ensure that the accuracy on classifying each stimulus was above 65%. The average lengths of height and width have been summarised in Table 2 and an example of the full set of stimuli is illustrated in Figure 2.

Table 2. Mean values of width and heights levels.

Dimensional level	Unbiased		Biased	
	Length (cm)	Visual angle (degree)	Length (cm)	Visual angle (degree)
Width base level (W_0)	7.67	4.08	7.73	4.12
Width low level ($W-$)	8.36	4.44	8.36	4.44
Width high level ($W+$)	8.91	4.74	8.78	4.67
Height base level (H_0)	7.67	5.5	7.73	5.54
Height low level ($H-$)	8.36	5.98	8.36	5.98
Height high level ($H+$)	8.73	6.26	8.63	6.17

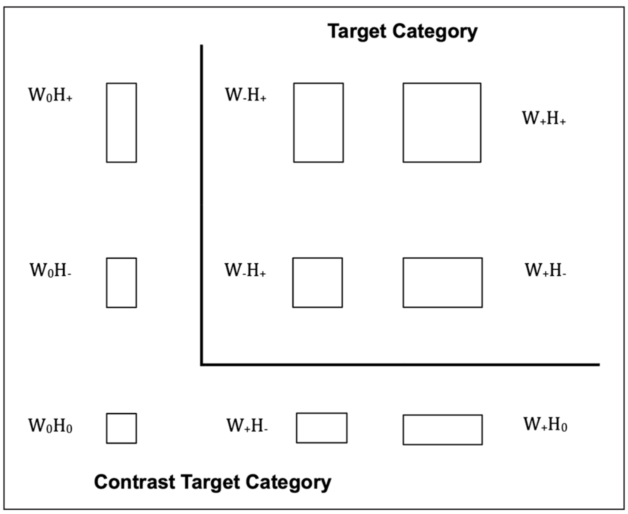

Figure 2. An exaggerated example of the full stimuli. The top-right four stimuli are used in the identification task, and then are assigned to the "target category" along with additional five stimuli from the "contrast target category" for classification.

Table 3. Seven block types of the identification task.

Block	Type	Fixed dimensional level	Stimuli
B1	Single dimension	($H-$)	$W-H-$, $W+H-$
B2	Single dimension	($H+$)	$W-H+$, $W+H+$
B3	Single dimension	($W-$)	$W-H-$, $W-H+$
B4	Single dimension	($W+$)	$W+H-$, $W+H+$
B5	Complete identification	None	All four rectangles
B6	Positively correlated	None	$W-H-$, $W+H+$
B7	Negatively correlated	None	$W-H+$, $W+H-$

All rectangular stimuli were generated using PsychoPy (Peirce et al., 2019) and displayed on monitors with 1920 × 1080 pixel resolutions. Each observer used a number keypad to make responses. The illuminance of all monitors were calibrated with a photometer on a black screen and were set to be 40 cd/m^2.

Design

This study involved two manipulated factors. The first factor was the type of tasks, which was either an identification task or a classification task. The identification task involved seven block types[5] (Table 3), four of which were single-dimensional block types and the rectangular stimuli for identification varied in only one dimension. The fifth block type was the complete identification block, where rectangular stimuli varied in both height and width. The last two were double-dimensional blocks with width changing along with height in either same (positively correlated) or opposite (negatively correlated) directions. The order of block types was randomised in each session of the identification task. As illustrated in Figure 2, in the identification task, there were a total of four rectangular stimuli composed of combinations of only low and high levels in each dimension.

In the classification task, the four rectangular stimuli from the identification task were assigned to the target category and additional five rectangular stimuli composed of combinations of a base level in either height or width and low/high levels in the other one were assigned to the contrast-target category (i.e., the double-factorial paradigm; see Supplementary Appendix A for the detailed description and Altieri et al., 2017).

The second factor concerned the payoff matrices (see Tables 4 and 5). In the unbiased condition, all responses received the same number of rewards, whereas in the two biased conditions (Biased 1 and Biased 2), a set of selected responses received more rewards for being correct and fewer penalties for being incorrect relative to the neutral responses. Payoffs for the favoured responses are in bold font in the payoff tables. Specifically, the payoffs in the Biased 1 condition objectively favoured the rectangular responses ($W-H+$, $W+H-$) in the complete identification task and the contrast-target response in the classification task; while the Biased 2 condition objectively favoured the square-looking responses ($W-H-$, $W+H+$) and the target

Table 4. Payoffs on responses in different biased conditions of the identification task.

Response	Reward (point)			Penalty (point)		
	Unbiased	Biased 1	Biased 2	Unbiased	Biased 1	Biased 2
W–H–	3	3	**9**	−1	−7	**−1**
W–H+	3	**9**	3	−1	**−1**	−7
W+H–	3	**9**	3	−1	**−1**	−7
W+H+	3	3	**9**	−1	−7	**−1**

Table 5. Payoffs on responses in different biased conditions of the classification task.

Response	Reward (point)			Penalty (point)		
	Unbiased	Biased 1	Biased 2	Unbiased	Biased 1	Biased 2
Target	5	1	**5**	−5	−5	**−1**
Contrast Target	5	**5**	1	−5	**−1**	−5

response in the identification and the classification tasks, respectively.

All the observers first did the identification and then the classification task with the unbiased payoff matrix. After, they did these two tasks again with one of the biased payoff matrix (either Biased 1 or Biased 2) that was randomly selected.

Procedure

Observers were tested individually in a dimly lit room and seated at a constrained viewing distance of 70 cm from the computer monitor using an adjustable chin rest. Each observer started the study with a calibration session to determine the magnitude of the high level on both the height and width dimensions. Then they started the identification task with the unbiased payoffs.

The identification task was consisted of four 1-hr sessions, each including a practice block of 40 trials and seven experimental blocks (see Table 3). At the beginning of each block, the set of stimuli for identification in the block appeared on the screen, along with their responding keys. The response key assigned to each stimulus was unique. Observers could take as much time as they wish to study the mapping of keys and stimuli. Once observers were ready, they proceeded to the actual identification task by pressing the space bar. On each trial, one of the stimuli appeared and observers were asked to make an identification judgement. The selection of a stimulus was randomised on each trial and the order of the blocks was randomised in each session. Each stimulus within an experimental block was presented 25 times. In each identification session, there were 100 trials in the complete identification block (B5) and 50 trials in other experimental blocks. In total, the identification task had 400 trials for the complete identification block and 200 trials for each single-trial block.

After observers completed all the identification sessions, they proceeded to the classification task with the unbiased payoffs. The classification task was consisted of three 1-hr sessions, each of which included seven blocks of 90 trials. At the beginning of each block, the full set of stimuli along with their assigned categories were presented for observers to study. The stimuli were classified into two categories. Once observers were ready, they proceeded to the actual classification task by pressing the space bar. On each trial, one of the stimuli was presented for observers to make a classification response. The selection of a stimulus was again randomised. Each classification stimulus was presented 10 times in each classification block. In total, observers completed a total of 1,890 trials in the classification task.

When observers completed both the identification and the classification task with the unbiased payoffs, they were randomly assigned to one of the two biased payoff conditions, and completed the two tasks again following the same procedures as they did in the unbiased condition. At the beginning of each task, the magnitudes of the dimensional levels were recalibrated for observers again to avoid ceiling effects.

The trial events of the identification and the classification sessions were identical. Each trial began with a fixation cross being presented for 500 ms at the centre of a monitor, which was then replaced by a rectangular stimulus that appeared for 250 ms. Observers had up to 2,000 ms to make an response once a stimulus was presented by pressing the corresponding key on the keyboard. Following each response, feedback was presented for 1,000 ms that informed observers of the accuracy of their response along with rewards earned for the trial and accumulated points over the session. There was a 500-ms interval between each trial. If observers failed to make a response, the programme would move on to the next trial. Data from missing trials were discarded prior to analyses.

Table 6. Comparisons of response proportions of reporting square-looking ones between the unbiased and biased condition.

Obs	Biased condition	p(square)		Biased—unbiased (%)	χ^2	p-value
		Unbiased (%)	Biased (%)			
O1	Biased 1	45.3	41.8	−3.5	1.84	.18
O2	Biased 1	45.8	36.0	−9.8	14.93	<.001***
O3	Biased 2	48.5	50.3	1.8	0.42	.52
O4	Biased 2	51.3	42.8	−8.5	11.23	<.001***
O5	Biased 2	46.8	52.5	5.8	5.08	.02*
O6	Biased 2	51.8	50.8	−1.0	0.12	.73
O7	Biased 1	49.0	42.0	−7.0	7.57	.01*
O8	Biased 1	47.5	46.8	−0.8	0.06	.80

MRI: magnetic resonance imaging.
Biased 1 favoured rectangular responses and Biased 2 favoured square responses.
***$p < .001$, *$p < .05$.

Results

Results of the complete identification task

The following analyses were performed on the data collected from the complete identification task for each observer. We first assessed cross-trial and within-trial independencies in response frequencies with non-parametric GRT measures including untimed and timed marginal response invariance (see Definitions 4 and 5 in Supplementary Appendix A) and untimed and timed report independence (see Definitions 6 and 7 in Supplementary Appendix A), respectively. Then, we presented analyses assessing the independencies between height and width from the perceptual and decisional aspects with parametric SDT measures including marginal d' and marginal decisional criterion (c). Finally, individualised multivariate Gaussian model comparisons (Thomas, 2001a, Macho, 2010) were conducted to confirm these inferences.

Results of response proportions as a function of payoffs. The response proportions were observed to change in the direction of payoff manipulations for most observers. In the unbiased condition, observers tended to be slightly biased towards rectangular responses over the square responses, as the response proportions of reporting square-looking stimuli were observed below 50% for most observers except for O4 and O6 (see Table 6). For those who later shifted to the biased condition where rectangular responses were objectively favoured (Biased 1), the response proportions of reporting squares were observed to decrease even further for all observers, indicating an increase in response bias towards rectangular patterns. For those who later shifted to the biased condition where square forms were objectively favoured (Biased 2), the response proportions of reporting squares were observed to increase significantly for only one (O5) out of four observers, indicating that most observers were reluctant to fully give up their preference for the rectangular shapes.

Results of static and timed marginal response invariance analyses. Marginal response invariance occurs if the response behaviour of each dimension remains the same when the value of the other dimension changes (i.e., cross-trial independence). We first considered the static marginal response invariance (see Definition 4 in Supplementary Appendix A) accounting only for response response frequencies. As presented in Tables 7 and 8, we compared the marginal response proportion of reporting a certain dimensional level on one dimension in the presence of low level of the other dimension versus the marginal response proportions in the presence of the high level of the other dimension using the z-score transformation test (Silbert & Thomas, 2013). A positive value in Tables 7 and 8 suggests a decrease in the marginal reporting proportion when the physical length of the other dimension increased, while a negative difference suggests an increase in the marginal reporting proportions with the increase in the physical length of the other dimension.

In the unbiased condition, it was found that the marginal proportions of reporting taller height ($H+$) was significantly larger when the stimulus width was narrow as opposed to when the stimulus width was wide for seven out of eight observers (see Table 7). Similarly, in the presence of short height, the marginal proportions of reporting wide width ($W+$) tended to be larger than those in the presence of tall height; the positive differences in the marginal reporting proportions of $W+$ were significant for six out of eight observers. In other words, the likelihoods of reporting the high level on a dimension were higher when observers were presented with the low level on the other dimension. By contrast, the marginal proportions of reporting low level on a dimension ($W-$ or $H-$) were observed to be smaller in the presence of the low physical level than in the presence of the high level on the other dimension. Half of observers (O1, O2, O4, and O5) significantly reported less $W-$ and $H-$ in the presence of low level of the other

Table 7. Differences of the marginal report proportions on each dimensional level when the other dimension shifted from low to high level and the results of *Z*-score Transformation Tests for *Marginal Response Invariance* (MRI) in the unbiased condition.

Obs	Reporting level	Difference	*Z*-score	*p*-value	MRI (Equation 1) Retained?
O1	*W*–	−0.228	−3.652	<.001***	No
	W+	0.283	5.299	<.001***	No
	H–	−0.439	−7.491	<.001***	No
	H+	0.407	7.042	<.001***	No
O2	*W*–	−0.212	−4.287	<.001***	No
	W+	0.32	5.984	<.001***	No
	H–	−0.263	−4.759	<.001***	No
	H+	0.45	8.547	<.001***	No
O3	*W*–	−0.01	−0.230	.8	Yes
	W+	0.13	3.058	.001**	No
	H–	−0.1	−1.808	.06	Yes
	H+	0.24	3.582	<.001***	No
O4	*W*–	−0.127	−2.424	.012*	No
	W+	0.06	1.418	.118	Yes
	H–	−0.18	−3.472	<.001***	No
	H+	0.152	2.198	.099	Yes
O5	*W*–	−0.11	−2.693	.003**	No
	W+	0.142	3.128	.001**	No
	H–	−0.17	−3.502	<.001***	No
	H+	0.202	4.127	<.001***	No
O6	*W*–	0.02	0.554	.517	Yes
	W+	0.08	1.588	.093	Yes
	H–	−0.02	−0.406	.663	Yes
	H+	0.17	2.972	.002**	No
O7	*W*–	−0.04	−1.178	.153	Yes
	W+	0.091	2.210	.015*	No
	H–	−0.04	−0.943	.297	Yes
	H+	0.122	2.650	.005**	No
O8	*W*–	0	0.000	1	Yes
	W+	0.1	2.119	.024*	No
	H–	−0.11	−2.298	.015*	No
	H+	0.19	3.281	.001**	No

MRI: magnetic resonance imaging.
****p* < .001, ***p* < .01, **p* < .05.

dimension. These patterns of the opposite changes in the marginal proportions of reporting low/high level on one dimension with the increase in the physical level of the other dimension would produce a reporting bias in favour of rectangles, which is consistent with our previous observations in response frequencies (Table 6).

In the biased conditions (Table 8) a similar pattern of marginal proportion variation was observed. For those who later shifted to the Biased 1 condition favouring *W*– *H*+ and *W*+ *H*– responses, three out of four observers (except for O8) continued to make significantly more high magnitude responses when the alternate dimension was at its lower level. For those who later shifted to the Biased 2 condition favouring *W*– *H*– and *W*+ *H*+ responses, all observers significantly reported either more *H*+ or more *W*+ in the presence of the low level of the other dimension; but only two out of four observers (O4 and O6) still made significantly more low-level responses on either height or width in the presence of high level of the other dimension.

Now, we turn to consideration of the temporal version of marginal response invariance (see Definition 5 in Supplementary Appendix A). When response times were taken into account, it was found that timed marginal response invariance[6] was significantly violated in both unbiased and biased conditions (see Tables 9 and 10). As expected, in those cases where the static analyses, marginal response invariance was violated (i.e., see O1 and O2 in the unbiased condition and O2 and O4 in the biased condition), the timed marginal response proportions of reporting a dimensional level were found to differ significantly across the physical lengths of the other dimension. These consistent observations of violations in both static and timed marginal response invariance were in agreement with the premises of the RTGRT theoretical framework (Figure 1b).

Table 8. Differences of the marginal report proportions on each dimensional level when the other dimension shifted from low to high level and the results of *Z*-score transformation tests for *marginal response invariance* (MRI) in biased conditions.

Obs	Biased condition	Reporting level	Difference	*Z*-score	*p*-value	MRI (Eq. 1) Retained?
O1	Biased 1	*W*–	−0.132	−2.953	.003**	No
		W+	0.080	1.588	.112	Yes
		H–	−0.439	−4.378	<.001***	No
		H+	0.250	4.085	<.001***	No
O2	Biased 1	*W*–	−0.190	−3.724	<.001***	No
		W+	0.200	3.705	<.001***	No
		H–	−0.263	−5.885	<.001***	No
		H+	0.300	5.345	<.001***	No
O3	Biased 2	*W*–	−0.050	−1.270	.204	Yes
		W+	0.010	0.385	.701	Yes
		H–	−0.100	0.287	.774	Yes
		H+	0.130	1.999	.046*	No
O4	Biased 2	*W*–	−0.120	−2.401	.016*	No
		W+	0.100	2.384	.017*	No
		H–	−0.180	−4.893	<.001***	No
		H+	0.280	4.787	<.001***	No
O5	Biased 2	*W*–	−0.070	−1.501	.133	Yes
		W+	0.150	3.168	.002**	No
		H–	−0.170	−1.478	.139	Yes
		H+	0.130	2.118	.034*	No
O6	Biased 2	*W*–	−0.070	−1.557	.120	Yes
		W+	0.100	2.645	.008**	No
		H–	−0.020	−2.500	.012*	No
		H+	0.080	1.303	.192	Yes
O7	Biased 1	*W*–	−0.062	−1.252	.211	Yes
		W+	0.150	2.683	.007**	No
		H–	−0.040	−4.008	<.001***	No
		H+	0.202	3.308	.001**	No
O8	Biased 1	*W*–	−0.070	−1.557	.120	Yes
		W+	0.060	1.952	.051	Yes
		H–	−0.110	−1.797	.072	Yes
		H+	0.080	1.443	.149	Yes

MRI: magnetic resonance imaging.
Biased 1 favoured rectangular responses, Biased 2 favoured square responses.
$^{***}p < .001$, $^{**}p < .01$, $^{*}p < .05$.

However, timed marginal response invariance of some observers failed even though static marginal response invariance was satisfied (i.e., see O3, O6, O7 at *W*– and *H*– and O4 at *W*+ in the unbiased condition; O3 and O8 in the biased conditions). This suggests that the processing times could, in some circumstances and for some observers, be a more sensitive measure for the non-separabilities than response frequencies.

The Kolmogorov–Smirnov tests (K-S test, Kolmogorov, 1933) on response times of reporting a certain dimensional level, supported our overall inferences. In both unbiased and biased conditions, the response times of reporting a certain dimensional level were found to differ significantly at the distributional level across the physical lengths of the other dimension for most observers.

In the unbiased condition (Table 11), the pattern of response time data suggests relatively faster identification at the dimensional level in the presence of rectangular stimuli than in the presence of the square ones. Recall that previously, the response frequencies of reporting rectangular ones were also observed (see Table 6). The joint observations of the relatively faster processing speed and the response biases towards rectangular ones is in alignment with the speed–accuracy trade-off. For those who later shifted to the Biased 1 condition favouring rectangle responses, a similar response-time pattern was observed (Table 12). By contrast, for those who later shifted to the Biased 2 condition favouring square responses, an opposite response-time pattern was observed. That is, the response times of reporting *W*– or *H*– were then found to be reliably faster in the presence of the low level of the

Table 9. Results of the Brownian-Bridge tests for *timed marginal response invariance* (tMRI) in the unbiased conditions.

Obs	Reporting level	$\hat{D}$	*p*-value	tMRI retained?
O1	*W*–	0.651	<.001***	No
	W+	0.772	<.001***	No
	H–	0.831	<.001***	No
	H+	0.741	<.001***	No
O2	*W*–	0.69	<.001***	No
	W+	0.878	<.001***	No
	H–	0.74	<.001***	No
	H+	0.849	<.001***	No
O3	*W*–	0.342	<.001***	No
	W+	0.666	<.001***	No
	H–	0.565	<.001***	No
	H+	0.662	<.001***	No
O4	*W*–	0.473	<.001***	No
	W+	0.526	<.001***	No
	H–	0.714	<.001***	No
	H+	0.518	<.001***	No
O5	*W*–	0.459	<.001***	No
	W+	0.601	<.001***	No
	H–	0.689	<.001***	No
	H+	0.618	<.001***	No
O6	*W*–	0.374	<.001***	No
	W+	0.453	<.001***	No
	H–	0.228	.010*	No
	H+	0.638	<.001***	No
O7	*W*–	0.366	<.001***	No
	W+	0.477	<.001***	No
	H–	0.229	.008**	No
	H+	0.67	<.001***	No
O8	*W*–	0.135	.194	Yes
	W+	0.494	<.001***	No
	H–	0.5	<.001***	No
	H+	0.5	<.001***	No

$^{***}p < .001$, $^{**}p < .01$, $^{*}p < .05$.

Table 10. Results of the Brownian-Bridge tests for *timed marginal response invariance* (tMRI) in the biased conditions.

Obs	Biased condition	Reporting level	$\hat{D}$	*p*-value	tMRI retained?
O1	Biased 1	*W*–	0.462	<.001***	No
		W+	0.450	<.001***	No
		H–	0.681	<.001***	No
		H+	0.645	<.001***	No
O2	Biased 1	*W*–	0.527	<.001***	No
		W+	0.730	<.001***	No
		H–	0.792	<.001***	No
		H+	0.714	<.001***	No
O3	Biased 2	*W*–	0.416	<.001***	No
		W+	0.269	.001**	No
		H–	0.476	<.001***	No
		H+	0.606	<.001***	No
O4	Biased 2	*W*–	0.419	<.001***	No
		W+	0.791	<.001***	No
		H–	0.775	<.001***	No
		H+	0.772	<.001***	No
O5	Biased 2	*W*–	0.244	.005***	No
		W+	0.500	<.001***	No
		H–	0.253	.007***	No
		H+	0.397	<.001***	No
O6	Biased 2	*W*–	0.465	<.001***	No
		W+	0.514	<.001***	No
		H–	0.561	<.001***	No
		H+	0.458	<.001***	No
O7	Biased 1	*W*–	0.433	<.001***	No
		W+	0.650	<.001***	No
		H–	0.699	<.001***	No
		H+	0.566	<.001***	No
O8	Biased 1	*W*–	0.309	<.001***	No
		W+	0.352	<.001***	No
		H–	0.333	<.001***	No
		H+	0.384	<.001***	No

Biased 1 favoured rectangular responses, Biased 2 favoured square responses.
$^{***}p < .001$, $^{**}p < .01$.

other dimension (except for O4). Moreover, the response times of reporting *W*+ and *H*+ were found to be either not significantly different across the other dimensional levels or even faster in the presence of high level of the other dimension (see O1 in Table 12). This opposite response-time pattern suggests that when the square-looking responses were objectively to be preferred, observers tended to be faster in identifying square stimuli than rectangular ones.

The results of the K-S tests suggest that the response times of reporting a dimensional level were reliably affected by the physical magnitude of the other dimension in the identification task. In addition, the introduction of the objective bias not only increased the response frequencies but also sped up the processing of the biased responses. Conjoining the inferences drawn from static and dynamic MRI analyses in this section, the inclusion of response-time measures further evidences the cross-trial marginal report dependency between height and width, and indicates potential violations of perceptual and decisional separability.

The statistically significant violation of marginal response invariance on height and width, both through response probabilities as well as response times, across the physical level of the other dimension confirms violations in perceptual separability and perhaps also decisional separability (see Figure 1a; also see Ashby & Townsend, 1986). That is, the information processing of height and width might be affected by the physical length of each other at both the perceptual and decisional phases. Subsequent assays based on sensitivity and bias parameters associated with the multidimensional theory of signal

Table 11. Results of the K-S tests for marginal speed invariance in the unbiased condition.

Obs	Reporting Level	$\Delta\overline{RT}$ (– vs.+)	K-S statistic	p-value	Speed Invariance?
O1	*W*–	15.822	0.138	.456	Yes
	W+	−281.724	0.598	<.001***	No
	H–	195.332	0.440	<.001***	No
	H+	−193.508	0.388	<.001***	No
O2	*W*–	168.149	0.539	<.001***	No
	W+	−169.900	0.558	<.001***	No
	H–	133.114	0.429	<.001***	No
	H+	−197.889	0.594	<.001***	No
O3	*W*–	28.018	0.122	.508	Yes
	W+	−93.168	0.280	.002**	No
	H–	−19.692	0.215	.043*	No
	H+	−114.120	0.372	<.001***	No
O4	*W*–	20.842	0.125	.482	Yes
	W+	−155.420	0.337	<.001***	No
	H–	151.645	0.291	.001**	No
	H+	32.561	0.101	.860	Yes
O5	*W*–	25.507	0.174	.122	Yes
	W+	−260.900	0.464	<.001***	No
	H–	134.607	0.399	<.001***	No
	H+	−154.932	0.298	.001**	No
O6	*W*–	7.892	0.142	.295	Yes
	W+	−57.981	0.261	.005**	No
	H–	19.362	0.158	.222	Yes
	H+	−31.865	0.208	.063	Yes
O7	*W*–	50.157	0.159	.152	Yes
	W+	−50.765	0.117	.558	Yes
	H–	−21.385	0.099	.705	Yes
	H+	−124.699	0.234	.013*	No
O8	*W*–	36.956	0.111	.635	Yes
	W+	−58.744	0.306	<.001***	No
	H–	75.536	0.230	.019*	No
	H+	−6.287	0.187	.110	Yes

***$p < .001$, **$p < .01$, *$p < .05$.

Table 12. Results of the K-S tests for marginal speed invariance in the biased conditions.

Obs (Cond.)	Reporting Level	$\Delta\overline{RT}$ (– vs.+)	Statistic	p-value	Speed invariance?
O1 (Biased 1)	*W*–	109.588	0.226	.021*	No
	W+	−204.503	0.396	<.001***	No
	H–	240.267	0.483	<.001***	No
	H+	−181.422	0.374	<.001***	No
O2 (Biased 1)	*W*–	156.821	0.334	<.001***	No
	W+	−156.292	0.343	<.001***	No
	H–	179.918	0.496	<.001***	No
	H+	−227.835	0.418	<.001***	No
O3 (Biased 2)	*W*–	−59.791	0.240	.007**	No
	W+	3.458	0.206	.027*	No
	H–	−225.346	0.511	<.001***	No
	H+	40.075	0.385	<.001***	No
O4 (Biased 2)	*W*–	7.863	0.112	.594	Yes
	W+	−192.637	0.487	<.001***	No
	H–	65.159	0.315	.001**	No
	H+	−168.333	0.344	<.001***	No
O5 (Biased 2)	*W*–	−85.089	0.264	.004**	No
	W+	−44.097	0.148	.256	Yes
	H–	−50.288	0.219	.041*	No
	H+	31.677	0.214	.062	Yes
O6 (Biased 2)	*W*–	−22.499	0.131	.422	Yes
	W+	−21.416	0.173	.120	Yes
	H–	−28.626	0.243	.017*	No
	H+	−7.786	0.156	.308	Yes
O7 (Biased 1)	*W*–	90.707	0.183	.094	Yes
	W+	−176.046	0.389	<.001***	No
	H–	151.301	0.413	<.001***	No
	H+	−91.079	0.225	.040*	No
O8 (Biased 1)	*W*–	−6.689	0.110	.592	Yes
	W+	−80.075	0.234	.008**	No
	H–	31.200	0.154	.232	Yes
	H+	−62.433	0.230	.020*	No

Biased 1 favoured rectangular responses, Biased 2 favoured square responses.
***$p < .001$, **$p < .01$, *$p < .05$.

detection (Ashby & Townsend, 1986; Kadlec & Townsend, 1992) and full-blown multidimensional models of signal recognition (Soto et al., 2015, 2017) will shine light on how to interpret the findings of the marginal report proportions.

Results of static and timed report independence. We now discuss the independence of response behaviour of each dimension within the specific stimulus patterns (i.e., within-trial independence). First, consider the static report independence that accounts only for the response frequencies (Definition 6 in Supplementary Appendix A). Comparisons of joint report proportions to the product of marginal report proportions on each dimension exhibited a reliable positive dependence between report proportions of height and width in the presence of square-looking stimuli (*W*– *H*– and *W*+ *H*+). In the unbiased condition (see Table 13), the results of χ^2 tests suggested that all observers' joint report proportions in the presence of *W*– *H*– and *W*+ *H*+ were significantly larger than the product of marginal report proportions on each dimension. When shifted to the biased conditions, the χ^2 results (see Table 14) suggested that in the presence of either *W*– *H*– and *W*+ *H*+ (Biased 2 condition), all observers' report proportions on height and width still tended to be positively dependent.

In stark contrast, in the presence of rectangular stimuli (*W*– *H*+ and *W*+ *H*–, Biased 1 condition), observers' report proportions on height and width tended towards independence. In the unbiased condition, six out of eight observers (except for O7 and O8) retained report independence on

Table 13. Results of Chi-square tests for *Report Independence* (RI) in the unbiased condition by comparing joint report proportions with the product of the marginals on height and width given each stimulus.

Obs	Stimulus	Difference	χ^2	*p*-value	RI (Equation 3) Retained?
O1	*W–H–*	0.146	36.3	<.001***	No
	W–H+	−0.008	0.935	.333	Yes
	W+H–	0.008	3.257	.071	Yes
	W+H+	0.128	29.303	<.001***	No
O2	*W–H–*	0.133	44.084	<.001***	No
	W–H+	0.000	0.038	.846	Yes
	W+H–	−0.002	0.192	.661	Yes
	W+H+	0.173	52.841	<.001***	No
O3	*W–H–*	0.050	17.928	<.001***	No
	W–H+	−0.008	0.54	.463	Yes
	W+H–	−0.002	0.273	.601	Yes
	W+H+	0.080	19.895	<.001***	No
O4	*W–H–*	0.140	64.391	<.001***	No
	W–H+	−0.013	1.019	.313	Yes
	W+H–	−0.003	0.331	.565	Yes
	W+H+	0.063	16.378	<.001***	No
O5	*W–H–*	0.093	45.789	<.001***	No
	W–H+	−0.001	0.058	.809	Yes
	W+H–	−0.001	0.124	.725	Yes
	W+H+	0.091	32.613	<.001***	No
O6	*W–H–*	0.025	14.16	<.001***	No
	W–H+	0.003	0.211	.646	Yes
	W+H–	−0.010	1.217	.27	Yes
	W+H+	0.092	30.01	<.001***	No
O7	*W–H–*	0.044	38.127	<.001***	No
	W–H+	0.009	11.135	.001**	No
	W+H–	−0.002	0.192	.661	Yes
	W+H+	0.071	34.883	<.001***	No
O8	*W–H–*	0.043	14.56	<.001***	No
	W–H+	0.039	17.263	<.001***	No
	W+H–	0.016	8.463	.004**	No
	W+H+	0.112	44.436	<.001***	No

***$p < .001$, **$p < .01$.

Table 14. Results of Chi-square tests for *Report Independence* (RI) in the biased condition by comparing joint report proportions with the product of the marginals on height and width given each stimulus.

Obs	Biased Condition	Stimulus	Difference	χ^2	*p*-value	RI (Equation 3) Retained?
O1	Biased 1	*W–H–*	0.026	1.972	.160	Yes
		W–H+	−0.004	0.457	.499	Yes
		W+H–	−0.016	2.166	.141	Yes
		W+H+	0.045	6.200	.013*	No
O2	Biased 1	*W–H–*	−0.002	0.013	.910	Yes
		W–H+	−0.003	0.331	.565	Yes
		W+H–	0.000	0.001	.981	Yes
		W+H+	0.043	4.034	.045*	No
O3	Biased 2	*W–H–*	0.048	12.004	.001**	No
		W–H+	−0.009	1.170	.279	Yes
		W+H–	−0.004	0.614	.433	Yes
		W+H+	0.003	0.202	.653	Yes
O4	Biased 2	*W–H–*	0.077	19.009	<.001***	No
		W–H+	−0.006	0.739	.390	Yes
		W+H–	0.009	9.796	.002**	No
		W+H+	0.062	14.531	<.001***	No
O5	Biased 2	*W–H–*	0.102	44.005	<.001***	No
		W–H+	−0.012	1.537	.215	Yes
		W+H–	−0.007	0.910	.340	Yes
		W+H+	0.073	16.487	<.001***	No
O6	Biased 2	*W–H–*	0.088	36.097	<.001***	No
		W–H+	−0.011	1.492	.222	Yes
		W+H–	−0.001	0.120	.729	Yes
		W+H+	0.060	18.841	<.001***	No
O7	Biased 1	*W–H–*	0.049	9.342	.002**	No
		W–H+	−0.002	0.035	.852	Yes
		W+H–	0.014	3.867	.049*	No
		W+H+	0.031	2.250	.134	Yes
O8	Biased 1	*W–H–*	0.043	14.523	<.001***	No
		W–H+	0.022	7.730	.005**	No
		W+H–	0.000	0.021	.885	Yes
		W+H+	0.037	14.946	<.001***	No

Biased 1 favoured rectangular responses, Biased 2 favoured square responses.
***$p < .001$, **$p < .01$, *$p < .05$.

height and width in the presence of both *W– H+* and *W+ H–*; whereas in the biased condition, five out eight observers (except for O4, O7 and O8) still evidenced report independence of height and width in the presence of these rectangular stimuli. For the few observers who violated report independence given rectangular stimuli, the χ^2 results suggest that their report proportions on height and width tend to depend on each other positively as in the square stimuli.

The results of the timed report independence (Tables 15 and 16) further reinforced the inferences drawn from the analyses of static report independence. See Definition 7 in Supplementary Appendix A for the formal definition of timed report independence and Supplementary Appendix B for the detailed analysis method. In both unbiased and biased conditions, the timed report proportions of height and width were reliably found to depend positively on each other in the presence of square-looking stimuli (*W– H–* and *W+ H+*) for all observers. In contrast, in the presence of the rectangular stimuli, observers were found to either retain the timed report independence (one in the unbiased condition and five out of eight in the biased conditions) or violated the timed report independence in an extremely weak positive or negative manner.

Altogether, when identifying rectangles, violations in timed report independence at a within-trial basis were observed consistently and were found to closely relate to

Table 15. Results of Brownian-Bridge tests for *timed report independence* (tRI) in the unbiased condition.

Obs	Stimulus	Avg. difference (joint—marginals)	*p*-value	tRI (Equation 4) Retained?
O1	*W–H–*	0.0587	<.001***	No
	W–H+	−0.0043	.032*	No
	W+H–	0.0039	.186	Yes
	W+H+	0.0667	<.001***	No
O2	*W–H–*	0.1187	<.001***	No
	W–H+	0.0000	1.000	Yes
	W+H–	−0.0010	<.001***	No
	W+H+	0.0676	<.001***	No
O3	*W–H–*	0.0481	<.001***	No
	W–H+	−0.0019	.313	Yes
	W+H–	−0.0014	.077	Yes
	W+H+	0.0204	<.001***	No
O4	*W–H–*	0.0328	<.001***	No
	W–H+	−0.0062	.232	Yes
	W+H–	−0.0015	.001**	No
	W+H+	0.0702	<.001***	No
O5	*W–H–*	0.0376	.005**	No
	W–H+	−0.0002	.193	Yes
	W+H–	−0.0003	.002**	No
	W+H+	0.0476	<.001***	No
O6	*W–H–*	0.0431	<.001***	No
	W–H+	−0.0004	.198	Yes
	W+H–	−0.0033	.031**	No
	W+H+	0.0145	<.001***	No
O7	*W–H–*	0.0417	<.001***	No
	W–H+	0.0031	.005**	No
	W+H–	−0.0006	.518	Yes
	W+H+	0.0255	<.001***	No
O8	*W–H–*	0.0451	<.001***	No
	W–H+	0.0131	.030*	No
	W+H–	0.0082	.078	Yes
	W+H+	0.0221	.085	Yes

***$p < .001$, **$p < .01$, *$p < .05$.

Table 16. Results of Brownian-Bridge tests for *timed report independence* (tRT) in the biased condition.

Obs	Biased condition	Stimulus	Avg. difference (joint—marginals)	*p*-value	tRI (Equation 4) retained?
O1	Biased 1	*W–H–*	0.018	.008**	No
		W–H+	−0.001	.370	Yes
		W+H–	−0.005	.261	Yes
		W+H+	0.010	.130	Yes
O2	Biased 1	*W–H–*	0.023	<.001***	No
		W–H+	−0.001	.045*	No
		W+H–	0.001	<.001***	No
		W+H+	−0.006	.261	Yes
O3	Biased 2	*W–H–*	0.002	.284	Yes
		W–H+	−0.005	.038*	No
		W+H–	−0.002	.392	Yes
		W+H+	0.025	<.001***	No
O4	Biased 2	*W–H–*	0.035	<.001***	No
		W–H+	−0.002	.392	Yes
		W+H–	0.005	.013*	No
		W+H+	0.039	<.001***	No
O5	Biased 2	*W–H–*	0.033	.014*	No
		W–H+	−0.008	.058	Yes
		W+H–	−0.002	.758	Yes
		W+H+	0.048	<.001***	No
O6	Biased 2	*W–H–*	0.032	<.001***	No
		W–H+	−0.004	.054	Yes
		W+H–	0.000	.156	Yes
		W+H+	0.051	<.001***	No
O7	Biased 1	*W–H–*	0.018	.058	Yes
		W–H+	0.001	.428	Yes
		W+H–	0.007	.219	Yes
		W+H+	0.025	.001**	No
O7	Biased 1	*W–H–*	0.016	.003**	No
		W–H+	0.005	.351	Yes
		W+H–	0.000	1.000	Yes
		W+H+	0.020	.158	Yes

Biased 1 favoured rectangular responses, Biased 2 favoured square responses.
***$p < .001$, **$p < .01$, *$p < .05$.

the shape of the stimuli in both unbiased and biased conditions. Observers were likely to violate report independence with a positive dependence in the presence of square patterns. In the presence of rectangular patterns, they showed much more inclination to independence.

Following the inductive logic of the GRT framework, the findings of report dependence between height and width infers that observers perceive height and width dependently especially in the presence of square-looking stimuli. In addition, their decisional criterion on each dimension might also be affected by the physical level of the other one. In other words, the findings of timed report dependency on height and width hints potential violations in perceptual independence and/or decisional separability on height and width when identifying rectangles.

In sum, results of static and dynamic GRT statistics indicate that all observers violated perceptual separability, perceptual independence and/or decisional separability of cognitive processes of identifying height and width of rectangles in both unbiased and biased conditions (see Figure 1).

As outlined in the introduction, the set of theoretical and methodological tools associated with GRT has evolved to permit three levels of analysis, the purely non-parametric (i.e., the observable response measure), the parametric (i.e., the signal detection measures), and full-model fitting procedures (i.e., the multidimensional signal detection models). We next assess various types of independence (or not) of the perceptual channels attending to height and width using parametric methods. To this end, we analyse the standard SDT measures, namely d's and decisional

Table 17. Results of marginal d' comparisons in the unbiased condition.

Obs	Dimension	d' comparisons	Observed difference	Z-score	p-value	Equality retained?
O1	W	$W\|H-$ vs $W\|H+$	2.056	5.866	<.001***	No
	H	$H\|W-$ vs $H\|W+$	0.239	0.760	.447	Yes
O2	W	$W\|H-$ vs $W\|H+$	1.261	3.034	.002**	No
	H	$H\|W-$ vs $H\|W+$	1.471	3.849	<.001***	No
O3	W	$W\|H-$ vs $W\|H+$	1.583	3.960	<.001***	No
	H	$H\|W-$ vs $H\|W+$	0.311	1.078	.281	Yes
O4	W	$W\|H-$ vs $W\|H+$	1.644	4.063	<.001***	No
	H	$H\|W-$ vs $H\|W+$	0.298	0.869	.385	Yes
O5	W	$W\|H-$ vs $W\|H+$	1.144	2.172	.030*	No
	H	$H\|W-$ vs $H\|W+$	0.630	1.652	.099	Yes
O6	W	$W\|H-$ vs $W\|H+$	1.027	2.182	.029*	No
	H	$H\|W-$ vs $H\|W+$	0.554	1.779	.075	Yes
O7	W	$W\|H-$ vs $W\|H+$	0.474	0.823	.410	Yes
	H	$H\|W-$ vs $H\|W+$	0.859	2.217	.027*	No
O8	W	$W\|H-$ vs $W\|H+$	0.863	2.282	.022*	No
	H	$H\|W-$ vs $H\|W+$	−0.054	−0.157	.876	Yes

***$p < .001$, **$p < .01$, *$p < .05$.

criteria c. We afterward conduct individualised Gaussian multi-dimensional signal detection model comparisons to affirm inferences of various kinds of dependencies.

Results of marginal d′ *analyses.* The comparison of marginal d's[7] on both dimensions of height and width indicated that perceptual sensitivity of each dimension was consistently affected by the physical levels of the other dimension.

The marginal d' s of width were found to be significantly different in the presence of different height levels for majority of the observers in both unbiased and biased payoff conditions (see Tables 17 and 18 for statistical results). In the unbiased condition, the perceptual sensitivity of width was significantly larger in the presence of shorter height than taller height for seven out of eight observers. When shifted to the biased condition, this general pattern in marginal d's on width was again observed. The perceptual sensitivity of width was significantly larger in the presence of short height than long height for one (O7) out of four in the Biased 1 condition favouring $W-$ $H+$ and $W-$ $H+$ responses, and half of observers (O4, O6) in the Biased 2 condition favouring $W-$ $H-$ and $W+$ $H+$ responses. This suggests that observers tended to be more sensitive to width differences when height was physically shorter.

The marginal d' s of height were observed to be larger in the presence of narrow ($W-$) rather than wide ($W+$) width, although the difference in these marginal d's was statistically significant among only a few observers. In the unbiased condition, the perceptual sensitivity of height was significantly larger in the presence of narrower width for two (O2, O7) out of eight observers given an unbiased payoff matrix and one (O3) out of eight given biased payoff matrices. These results suggest that observers tended to be more sensitive to height differences when width was physically narrow. Moreover, the effects of height and width on each other's perceptual sensitivity were asymmetric. The physical length of height turned to have a more prevalent and stronger affect on the perceptual sensitivity of width than width on height, as indicated by the relatively larger differences in marginal d's on width than on height.

The unequal marginal d's of width and height in the presence of different length of the other dimension suggest that when identifying rectangles, observers' perceptual sensitivity was affected by the physical magnitude of the other dimension. In the presence of a shorter height or a narrower width, observers tended to perceive the difference in the other dimension better. The observed unequal marginal d's on each dimension further suggested that observers do in fact violate perceptual separability when identifying rectangles. That is, the magnitude of width influences the percept of height, and the magnitude of height influences the percept of width. In addition, they do so in a way that is completely in line with the earlier violations of marginal response invariance.

Results of marginal decisional criteria analyses. The comparisons of marginal decisional criteria find that the marginal decisional bounds on height and width differ with the physical magnitude of each other. Moreover the shift of marginal decisional criteria, when they occur, are generally in the expected direction based on the payoff conditions.

First, consider the unbiased condition (Table 19) wherein payoffs were assigned neutrally for each stimulus. The observed difference in marginal decisional criteria on height and on width increased with the increase in the length of the other dimension (as the difference of marginal c's is less than 0) for all observers but only some (O1, O2, O5, O8) were the changes significant. This

Table 18. Results of marginal *d*′ comparisons in the biased condition.

Obs	Biased condition	Dimension	*d*′ comparisons	Difference	Z-score	*p*-value	Equality Retained?
O1	Biased 1	W	*W|H– vs W|H+*	0.131	0.354	.724	Yes
		H	*H|W– vs H|W+*	0.277	1.001	.317	Yes
O2	Biased 1	W	*W|H– vs W|H+*	0.347	1.059	.290	Yes
		H	*H|W– vs H|W+*	0.354	1.217	.224	Yes
O3	Biased 2	W	*W|H– vs W|H+*	0.562	1.050	.294	Yes
		H	*H|W– vs H|W+*	0.497	2.144	.032*	No
O4	Biased 2	W	*W|H– vs W|H+*	−0.302	3.907	<.001***	No
		H	*H|W– vs H|W+*	1.445	−0.791	.429	Yes
O5	Biased 2	W	*W|H– vs W|H+*	0.559	0.000	1.000	Yes
		H	*H|W– vs H|W+*	5.972	1.951	.051	Yes
O6	Biased 2	W	*W|H– vs W|H+*	0.021	2.950	.003**	No
		H	*H|W– vs H|W+*	1.393	0.070	.944	Yes
O7	Biased 1	W	*W|H– vs W|H+*	1.147	3.394	.001**	No
		H	*H|W– vs H|W+*	−0.129	−0.388	.698	Yes
O8	Biased 1	W	*W|H– vs W|H+*	0.463	1.145	.252	Yes
		H	*H|W– vs H|W+*	0.058	0.165	.869	Yes

Biased 1 favoured rectangular responses and Biased 2 favoured square responses.
$^{***}p < .001$, $^{**}p < .01$, $^{*}p < .05$.

Table 19. Results of marginal decisional criterion comparisons in the unbiased condition.

Obs	Dimension	*c* comparisons	Observed difference	*Z*-score	*p*-value	Equality Retained?
O1	W	*W|H– vs W|H+*	−0.433	−1.193	.233	Yes
	H	*H|W– vs H|W+*	−0.467	−3.124	.002**	No
O2	W	*W|H– vs W|H+*	−0.378	−2.270	.023*	No
	H	*H|W– vs H|W+*	−0.560	−3.462	.001**	No
O3	W	*W|H– vs W|H+*	−0.296	−0.807	.420	Yes
	H	*H|W– vs H|W+*	−0.276	−1.376	.169	Yes
O4	W	*W|H– vs W|H+*	−0.266	−0.841	.400	Yes
	H	*H|W– vs H|W+*	−0.087	−0.253	.801	Yes
O5	W	*W|H– vs W|H+*	−0.161	−0.902	.367	Yes
	H	*H|W– vs H|W+*	−0.294	−2.509	.012*	No
O6	W	*W|H– vs W|H+*	−0.034	0.581	.562	Yes
	H	*H|W– vs H|W+*	−0.149	−0.953	.340	Yes
O7	W	*W|H– vs W|H+*	−0.093	−0.668	.504	Yes
	H	*H|W– vs H|W+*	−0.140	−1.152	.249	Yes
O8	W	*W|H– vs W|H+*	−0.181	−1.225	.221	Yes
	H	*H|W– vs H|W+*	−0.217	−2.031	.042*	No

$^{**}p < .01$, $^{*}p < .05$.

suggests that if the width is wide, then observers are less likely to respond "tall" than if the width is narrow. And, if the height is tall, observers are less likely to respond "wide" than if the height is short. Such a pattern indicates that, on the width dimension, the response region of *W– H+* was larger than that of *W– H–*, and the response region of *W+ H+* was smaller than that of *W– H+*. On the height dimension, the response region of *W+ H–* was larger than that of *W– H–* but the response region of *W+ H+* was smaller than that of *W+ H–*. Together, these effects point to an overall preference for rectangular over square figures, even when the payoffs assigned for different stimuli were objectively unbiased.

For the observers (O1, O2, O7 and O8) who later shifted to the Biased 1 condition favouring *W– H+* and *W+ H–* responses, the differences in marginal criteria on both dimensions were again observed to be negative. That is, the marginal criterion on each dimension increased with the increase in the length of the other dimension. Such a pattern subsequently implies response biases towards rectangular forms, which is consistent with the objective manipulation of the payoffs the in the Biased 1 condition and the observed change in response frequencies (Table 6). The statistical results (Table 20) further affirm that, accounting for both dimensions, all observers' marginal decisional criteria on at least one dimension were

Table 20. Results of marginal criteria comparisons in the biased condition.

Obs	Biased condition	Dimension	c comparisons	Difference	Z-score	p-value	Equality Retained?
O1	Biased 1	W	$W\|H-$ vs $W\|H+$	−0.209	−1.625	.104	Yes
		H	$H\|W-$ vs $H\|W+$	−0.599	−3.217	.001**	N
O2	Biased 1	W	$W\|H-$ vs $W\|H+$	−0.402	−2.771	.006**	No
		H	$H\|W-$ vs $H\|W+$	−0.793	−4.418	<.001***	No
O3	Biased 2	W	$W\|H-$ vs $W\|H+$	0.063	1.155	.248	Yes
		H	$H\|W-$ vs $H\|W+$	0.015	−0.018	.986	Yes
O4	Biased 2	W	$W\|H-$ vs $W\|H+$	−0.355	−2.277	.023*	No
		H	$H\|W-$ vs $H\|W+$	−0.370	−3.381	.001**	No
O5	Biased 2	W	$W\|H-$ vs $W\|H+$	0.097	0.000	1.000	Yes
		H	$H\|W-$ vs $H\|W+$	−0.104	−0.316	.752	Yes
O6	Biased 2	W	$W\|H-$ vs $W\|H+$	−0.073	0.252	.801	Yes
		H	$H\|W-$ vs $H\|W+$	−0.064	−0.498	.619	Yes
O7	Biased 1	W	$W\|H-$ vs $W\|H+$	−0.359	−1.763	.078	Yes
		H	$H\|W-$ vs $H\|W+$	−0.427	−3.352	.001**	No
O8	Biased 1	W	$W\|H-$ vs $W\|H+$	−0.235	−2.132	.033*	No
		H	$H\|W-$ vs $H\|W+$	−0.082	−0.773	.440	Yes

Biased 1 favoured rectangular responses, Biased 2 favoured square responses.
***$p < .001$, **$p < .01$, *$p < .05$.

significantly different across the physical level of the other dimension. Thus, the tendencies of preferring rectangles over squares towards which observers were inclined in the neutral bias condition were amplified in the bias-towards-rectangle condition.

For observers (O3, O4, O5, O6) who later shifted to the Biased 2 condition objectively favouring square-looking responses (see Table 20), the change in marginal criteria was observed to shift consistent with the objective manipulation of payoffs. Specifically, the differences in marginal criteria were observed to be positive on the width dimension for half of observers (O3, O5), and on the height dimension for one observer (O3). This suggests that pair-wise on the width dimension, the response region of *W*+ *H*+ was larger than that of *W*+ *H*–, and the response region of *W*– *H*– was larger than that of *W*– *H*+. In fact, decreases in response frequencies of reporting squares were observed for some observers in the Biased 2 condition (Table 6), which is in agreement with these inferences drawn from the shifts in estimated marginal criteria. However, our statistical analyses suggest that the difference in marginal criteria on both dimensions were not significant for most of the observers, excepting O4. The payoff changes moved people in the expected direction but not enough to completely overpower the preference for rectangles over squares.

In sum, the effect of the physical length of one dimension on the decisional criterion of the other dimension was found to be associated with the type of payoff manipulations, with some fairly minor individual differences. Moreover, these analyses uncovered a decided overall failure of decisional separability with a marked response bias towards rectangular stimuli. And, the payoff manipulations, while militating against that already present partiality towards rectangles were not able to completely overcome that preference.

Fitting multivariate Gaussian models to individual data. To further confirm the inferences drawn so far, we fit a hierarchy of multivariate Gaussian models to the identification/confusion matrices of individual observers.[8] First, we remind the reader that violations of perceptual separability or decisional separability or both can result in violations in marginal report invariance and violations in perceptual independence or decisional separability or both can result in violations of report independence.

The findings drawn from the marginal and joint report proportions and marginal SDT measures can deductively guide us to infer possible kinds of underlying dependencies (see Figure 3 in Wenger & Rhoten, 2020) but now within the context of fitting individual data with a set of candidate multivariate Gaussian models. We then selected the best-fitting model based upon the AIC score (Akaike, 1974) to compare with the earlier analyses.

In the interest of space, we begin with an illustration for a single observer and then move on to relate the global findings. Consider the results of O1 in the unbiased condition. We determined first that marginal response invariance of this observer (Table 7) was violated. Next, the marginal d' on width was unequal across different magnitudes of height with the shorter height being associated with a larger d' (Table 17). Moreover, the marginal criterion on height was a bit smaller for a small width (Table 19), suggesting a bias towards the narrow and tall rectangle (*W*– *H*+) vs. the small square (*W*– *H*–) and opposed to the big square (*W*+ *H*+) vs. the short and wide rectangle (*W*+ *H*–). These findings suggest that violations in both

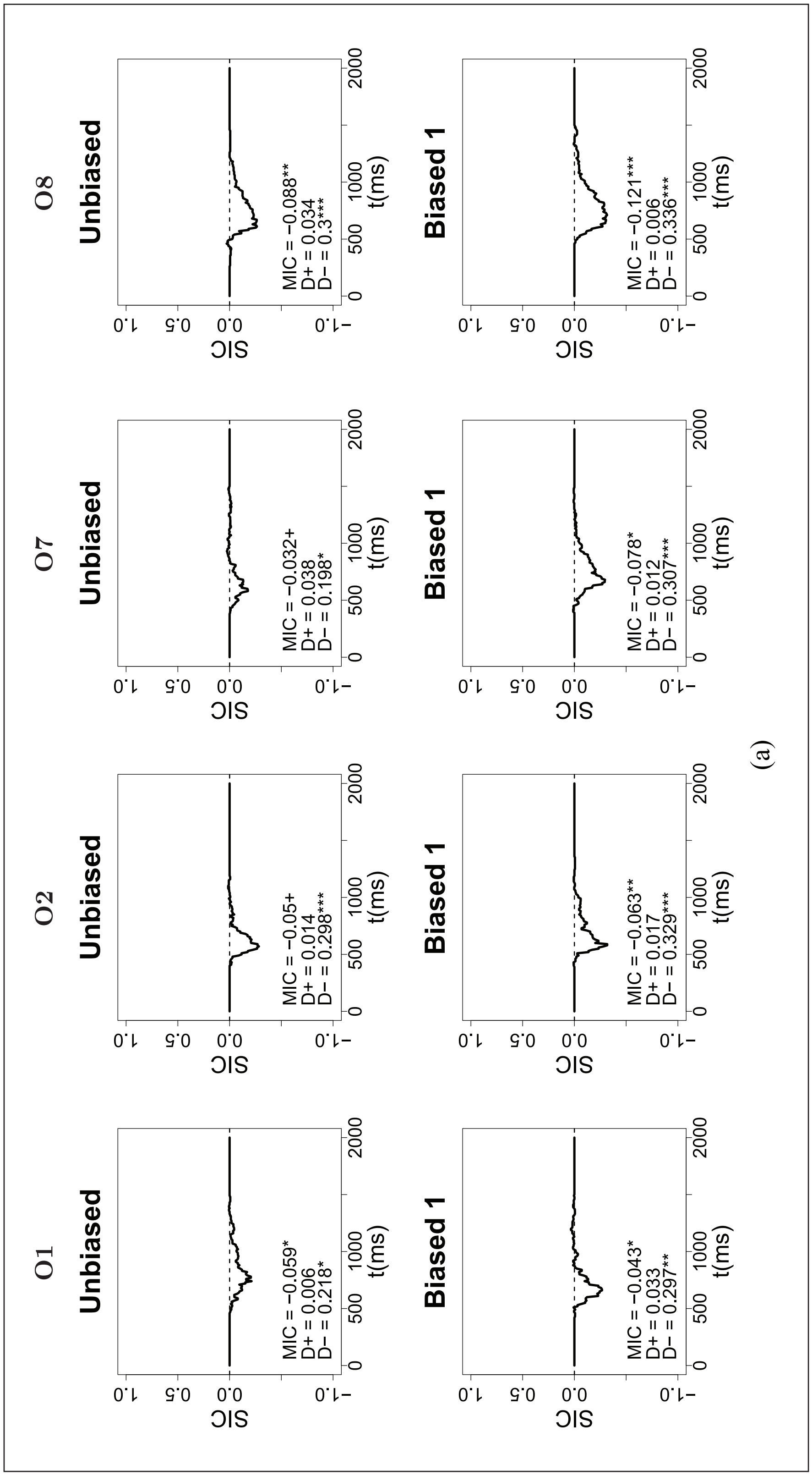

Figure 3. (continued)

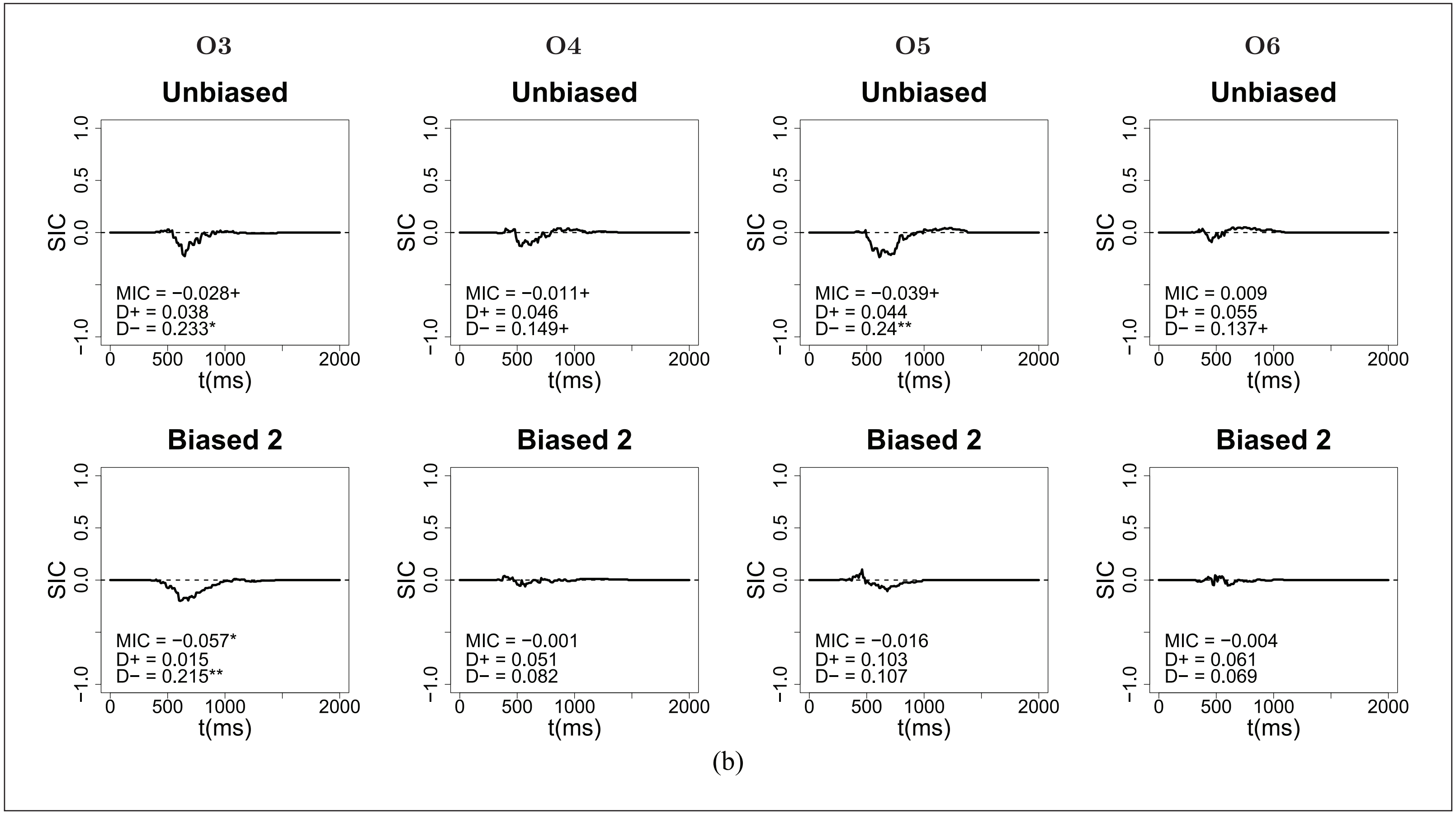

Figure 3. (a) Plots of estimated *SIC*(*t*) and *MIC* values along with indications of statistical significance for observers completed an unbiased condition and the Biased 1 condition favouring contrast-target responses. (b) Plots of estimated *SIC*(*t*) and *MIC* values along with indications of statistical significance for observers completed an unbiased condition and the Biased 2 condition favouring target responses. D^+ denotes the absolute value of estimated supremum positive deviation from zero of *SIC*(*t*); D^- denotes the absolute value of estimated supremum negative deviation from zero of *SIC*(*t*).
$^{***}p < .001$, $^{**}p < .01$, $^{*}p < .05$.

Table 21. Results of individualised Gaussian multidimensional model comparisons in the unbiased condition.

Obs	Best-fitted model	PS retained?	PI retained?	DS retained?
O1	PI(W-H +, W + H-) + DS(W)	No	on W-H +, W + H-	on Width
O2	PI(W-H +, W + H-)	No	on W-H +, W + H-	No
O3	PI(W-H +, W + H-) + DS	No	on W-H +, W + H-	Yes
O4	PI(W-H +, W + H-) + DS	No	on W-H +, W + H-	Yes
O5	PI(W-H +, W + H-) + DS(W)	No	on W-H +, W + H-	on Width
O6	PI(W-H +, W + H-) + DS	on Height	on W-H +, W + H-	Yes
O7	DS	No	No	Yes
O8	PS(H) + DS(W)	on Height	No	on Width
Total Violations		8	8	4

PS: perceptual separability; PI: perceptual independence; DS: decisional separability; W: width; H: height; −: low level; +: high level.

Table 22. Results of individualised Gaussian multidimensional model comparisons in the biased condition.

Obs	Biased condition	Best-fitted model	PS retained?	PI retained?	DS Retained?
O1	Biased 1	PI + DS(W)	No	Yes	on Width
O2	Biased 1	PI(W-H +, W + H-) + DS(W)	No	on W-H +, W + H-	on Width
O3	Biased 2	PS(H) + DS + PI(W-H +, W + H-, W + H +)	on Height	on W-H +, W + H-, W + H +	Yes
O4	Biased 2	PI(W-H +)	No	on W-H +	No
O5	Biased 2	DS + PI(W-H +, W + H-)	No	on W-H +, W + H-	Yes
O6	Biased 2	DS + PI(W-H +, W + H-)	No	on W-H +, W + H-	Yes
O7	Biased 1	PI(W-H +, W + H +) + DS(W)	No	on W-H +, W + H +	on Width
O8	Biased 1	PI(W + H-) + PS + DS	Yes	on W + H-	Yes
Total Violations			7	7	4

PS: perceptual separability; PI: perceptual independence; DS: decisional separability; W: width; H: height; −: low level; +: high level.
Biased 1 favoured rectangular responses, Biased 2 favoured square responses.

perceptual and decisional separability contribute to failure of the marginal response invariance for this observer.

In addition, on the key within-trial statistic, the report independence was also found to be strongly violated on the square stimuli, indicating O1 may violate perceptual independence between height and width when confronted with square patterns. However, because decisional separability appears to fail on width, it is reasonable to propose a set of multivariate Gaussian models[9] that allow violations in perceptual separability on width, decisional separability on height while leaving open the issue of perceptual independence, in the presence of square-looking stimuli.

The results of individualised model comparisons affirmed the effect of physical settings of height and width on the percepts of each other in both unbiased and biased conditions. In the unbiased condition (see Table 21), the best-fitting models of all observers violated perceptual separability on width. Moreover, six out of eight (except for O6 and O8) also violated perceptual separability on height. In the biased condition, similar patterns were observed (see Table 22). For majority of the observers, the best-fitting models again were found to violate perceptual separability on width and height.

Moving to perceptual independence, the best-fitting models exhibited very strong correlations given square-looking stimuli, which is consistent with our inferences drawn from the report independence analyses. In the unbiased condition, the best-fitting models of all observers violated perceptual independence in the presence of *W− H−* and *W+ H+*; whereas in the presence of rectangular ones, the best-fitting models of six out of eight observers (except for O7 and O8) retained perceptual independence. In the biased conditions, the best-fitting models again suggested similar patterns of perceptual dependence between the percepts of height and width within square-looking stimuli yet perceptual independence within rectangular ones, except for O1.

With respect to decisional separability, the most decisive failures in decisional separability found in the earlier analyses of marginal criteria were corroborated by our model fits. And there were substantial individual differences regarding decisional separability. In the unbiased condition, half of the observers (O1, O2, O5, and O8) were inferred to violate decisional separability on the height dimension, while the best-fitting models of the other half still retained the decisional separability on both dimension. For those (O1, O2, O7 and O8) who later shifted to the Biased 1 condition favouring *W− H+* and *W+ H−* responses, similar patterns of violations of decisional separability on width but retained on height were again inferred for three out of four observers. For those (O3, O4, O5 and O6) who shifted to the Biased 2 condition favouring *W*– *H*– and *W*+ *H*+ responses, decisional separability was inferred to hold on both dimension for majority of the observers.

In sum, the results of fitting individualised multivariate Gaussian models strongly reinforced the inferences obtained with the non-parametric MRI and RI tests, and the estimated SDT sensitivity and bias parameters. Moreover, the underlying positive dependency between the percepts of height and width in squares (not rectangles) were reinforced across observers in the different payoff conditions. Although we found more individual differences in the decision bounds accompanying the model fits, in general, they tended to mirror our earlier inferences with estimated marginal decisional criteria.

Conclusion. This phase of our study took the rare step of detailed tests at three echelons of analysis from the quite macroscopic level of nonparametric measures, to general signal detection parameters, to Gaussian model fits to individual data. The three levels of analysis agree in unveiling a consistent picture of configurality in the way that perception and decision regarding length and width interact. Sensory perception of either dimension was more accurate when the other dimension was less in magnitude. There was a decisional preference for rectangles over squares. And, the perceived dimensions were positively correlated (within-trial basis) in squares but not rectangles.

Results of the classification task

The following analyses were performed on data collected from the classification task in the presence of stimuli from the target category. As noted previously, all stimuli in the target category in the classification task were the same as the set used in the complete identification task. These conditions permit us to assess architecture, decisional stopping rule, and workload capacity.

We start by noting that the accuracy rates of all observers were above 85%. Tables 23 and 24 summarise the SDT statistics. The estimated *d*′s for each observer support the overall constancy of perceptual sensitivity across unbiased and biased conditions. In addition, the estimated decisional criteria changed with the payoff manipulations as expected. That is, when shifted from the unbiased to the Biased 1 condition favouring contrast-target responses, the estimated criteria increased, suggesting a bias towards contrast stimuli and away from the targets. In contrast, when the payoff condition shifted from unbiased to the Biased 2 condition favouring target responses, criteria decreased, suggesting an augmented bias in favour of target responses.

Our initial tests aim at the mental architectures and stopping rules of the underlying cognitive process, utilising $SIC(t)$ functions (see Definition 9 in Supplementary Appendix A; statistically per Houpt & Townsend, 2010), and *MIC* (see Definition 8 in Supplementary Appendix A; statistically tested

Table 23. Estimated SDT statistics of responses from unbiased and biased 1 conditions.

Os	d′		Criterion	
	Unbiased	Biased 1	Unbiased	Biased 1
O1	2.175	2.066	0.071	0.308
O2	2.856	2.493	0.381	0.312
O7	3.021	2.802	0.174	0.283
O8	2.765	2.091	0.141	0.306

Table 24. Estimated SDT statistics of responses from unbiased and Biased 2 conditions.

Os	d′		Criterion	
	Unbiased	Biased 2	Unbiased	Biased 2
O3	2.359	3.467	0.029	−0.268
O4	2.359	2.359	0.164	−0.069
O5	2.180	2.264	0.240	−0.047
O6	3.079	3.105	0.245	0.079

with the adjusted rank-transform test) estimated from correct response times. Then, we examined workload capacity with capacity coefficients ($C(t)$; see Definition 10 in Supplementary Appendix A) estimated from response times only and $A_{CF}(t)$ accounting for both response time and response accuracy. All the analyses were conducted using the "SFT" R package (Houpt et al., 2014); trials with extreme response times (below 200 ms or above 1500 ms) were discarded before analyses.

Survivor and mean interaction contrast results. The estimated $SIC(t)$ and MIC statistics are shown in Figure 3a and b. All observers passed the Kolmogorov–Smirnov tests (Kolmogorov, 1933) for stochastic dominance of survival functions (Tables 25 and 26). In other words, the response times in the presence of stimuli from target category were significantly ordered by the combinations of dimensional salience levels. Thus, the data do not violate the vital assumption of selective influence at the level of distributional ordering (Townsend, 1990).

The results of SIC and MIC analyses offer overwhelming evidence that, in the unbiased condition, all observers (except for O6) processed height and width following an exhaustive-parallel stopping rule in the presence of rectangles. A negative $SIC(t)$ in conjunction with a negative MIC were obtained for each of these observers. This finding confirms that, when presented with rectangles from the target category, observers tended to process height and width simultaneously and a response was made after the completion of processing on both dimensions. O6, in contrast to the parallel architecture of all others, tended to follow a serial exhaustive mental architecture, as indicated by the corresponding estimated $SIC(t)$ and $MIC(=0.009)$.

Table 25. Results of K-S tests for examining the stochastic dominance in RT survivor functions of those who shifted from unbiased to Biased 1 condition.

Obs	Comparison	$\hat{D}$	
		Unbiased	Biased 1
O1	$W+H+ > W-H+$	0.179**	0.185**
	$W+H+ > W+H-$	0.299***	0.315***
	$W-H+ > W-H-$	0.169*	0.130+
	$W+H- > W-H-$	0.046	0.056
	$W+H+ < W-H+$	0.000	0.029
	$W+H+ < W+H-$	0.005	0.032
	$W-H+ < W-H-$	0.035	0.038
	$W+H- < W-H-$	0.084	0.132
O2	$W+H+ > W-H+$	0.290***	0.251***
	$W+H+ > W+H-$	0.440***	0.282***
	$W-H+ > W-H-$	0.195**	0.072
	$W+H- > W-H-$	0.133+	0.071
	$W+H+ < W-H+$	0.000	0.000
	$W+H+ < W+H-$	0.000	0.017
	$W-H+ < W-H-$	0.006	0.076
	$W+H- < W-H-$	0.064	0.082
O7	$W+H+ > W-H+$	0.221***	0.307***
	$W+H+ > W+H-$	0.467***	0.399***
	$W-H+ > W-H-$	0.352***	0.183**
	$W+H- > W-H-$	0.078	0.084
	$W+H+ < W-H+$	0.014	0.005
	$W+H+ < W+H-$	0.000	0.006
	$W-H+ < W-H-$	0.006	0.000
	$W+H- < W-H-$	0.028	0.011
O8	$W+H+ > W-H+$	0.265***	0.273***
	$W+H+ > W+H-$	0.316***	0.350***
	$W-H+ > W-H-$	0.100	0.047
	$W+H- > W-H-$	0.029	0.031
	$W+H+ < W-H+$	0.033	0.000
	$W+H+ < W+H-$	0.000	0.000
	$W-H+ < W-H-$	0.053	0.030
	$W+H- < W-H-$	0.060	0.133

$^{***}p < .001$, $^{**}p < .01$, $^{*}p < .05$, $^{+}p < .1$.

Table 26. Results of K-S tests for examining the stochastic dominance in RT survivor functions of those who shifted from unbiased to Biased 2 condition.

Obs	Comparison	$\hat{D}$	
		Unbiased	Biased 2
O3	$W+H+ > W-H+$	0.194**	0.113
	$W+H+ > W+H-$	0.234***	0.236***
	$W-H+ > W-H-$	0.125	0.097
	$W+H- > W-H-$	0.094	0.015
	$W+H+ < W-H+$	0.016	0.012
	$W+H+ < W+H-$	0.015	0.010
	$W-H+ < W-H-$	0.039	0.022
	$W+H- < W-H-$	0.069	0.118
O4	$W+H+ > W-H+$	0.137*	0.135*
	$W+H+ > W+H-$	0.212***	0.214***
	$W-H+ > W-H-$	0.100	0.170**
	$W+H- > W-H-$	0.085	0.112
	$W+H+ < W-H+$	0.024	0.018
	$W+H+ < W+H-$	0.024	0.026
	$W-H+ < W-H-$	0.015	0.010
	$W+H- < W-H-$	0.043	0.000
O5	$W+H+ > W-H+$	0.193***	0.079
	$W+H+ > W+H-$	0.486***	0.326***
	$W-H+ > W-H-$	0.297***	0.303***
	$W+H- > W-H-$	0.070	0.067
	$W+H+ < W-H+$	0.023	0.070
	$W+H+ < W+H-$	0.005	0.017
	$W-H+ < W-H-$	0.000	0.010
	$W+H- < W-H-$	0.081	0.070
O6	$W+H+ > W-H+$	0.087	0.057
	$W+H+ > W+H-$	0.134*	0.136*
	$W-H+ > W-H-$	0.115	0.137*
	$W+H- > W-H-$	0.089	0.064
	$W+H+ < W-H+$	0.049	0.056
	$W+H+ < W+H-$	0.020	0.038
	$W-H+ < W-H-$	0.032	0.028
	$W+H- < W-H-$	0.075	0.035

$^{***}p < .001$, $^{**}p < .01$, $^{*}p < .05$, $^{+}p < .1$.

When observers shifted to the biased conditions, those who participated in the Biased 1 condition favouring contrast stimuli preserved the *SIC*(t) patterns (see Figure 3a) obtained in the unbiased condition. Interestingly, in contrast, of those observers who switched to the Biased 2 condition (see Figure 3b) favouring the target responses, only O3 evidenced exhaustive parallel processing.

From the standpoint of canonical *SIC*(t) functions (e.g., Townsend & Nozawa, 1995) the obtained *SIC*(t) curves might seem to suggest that the stopping rule followed by some observers (O4, O5 and O6) assigned to the Biased 2 condition shifted to employ a self-terminating stopping rule. Nevertheless, if these observers truly employed a self-terminating stopping rule, then we would expect to observe a dramatic drop in accuracy with the presence of target stimuli. This is because the experimental design naturally enforced a conjunctive stopping rule for target stimuli. The empirical evidence does not support this hypothesis. Similar accuracy levels of these observers were observed across unbiased (88%, 85%, 91% with respect to O4, O5 and O6) and positively biased condition (88%,86%, 93%).

Another possibility is that observers may mix the use of different stopping rules or/and different mental architectures in the Biased 2 condition favouring target responses. Recall that to correctly classify rectangles from the target category, a conjunctive stopping rule is required; while to correctly classify rectangles from the contrast target category, a disjunctive stoppling rule is sufficient. When observers were assigned to the Biased 1 condition where an incorrect target-response on contrast-target stimuli would lead to a strong penalty, observers might have been more conservative in their responses towards the target stimuli, so that they were most likely to employ a conjunctive stopping rule. The inferences drawn from the estimated $SIC(t)$ suggesting a parallel-exhaustive system are consistent with this hypothesis.

By contrast, when observers were assigned to the Biased 2 condition where an incorrect target response would incur less of a penalty compared with an incorrect contrast-target response, it was likely that observers might have been less conservative in their responses towards target stimuli sometimes and thus led to a mixed use of conjunctive and disjunctive stopping rules in the presence of target stimuli. However, the testing of this post hoc hypothesis calls for extension of the theoretical frame of SFT to incorporate systems with hybrid stopping rules, which to our knowledge has not been fully investigated yet.

Capacity coefficient results. Recall that the capacity coefficient is a ratio-function comparing response time performance from double-channel conditions with those from single-channel conditions (Definition 10). In the current study, the numerator of the capacity coefficients was estimated from the response times of target stimuli collected in the classification task and the denominator was estimated from the response times collected in single-dimensional blocks from the identification task where observers were explicitly instructed to focus on only a single dimension to make a response.

Estimated capacity coefficients $C(t)$ (Figure 4a and b) were assessed using the methods of Houpt and Burns (2017) and were found to be significantly above the benchmark of an unlimited-capacity independent parallel model (UCIP). Thus, observers here appear to be endowed with super capacity: the engagement in processing both height and width evidenced processing efficiency as measured by the processing speed that was superior to standard, unlimited capacity systems. Moreover, the $C(t)$s of all observers were even above the Colonius-Vorberg upper bounds (C-V bound, Colonius & Vorberg, 1994). This demarcates a level of performance designated as extreme super capacity (see e.g., Townsend et al., 2020; Townsend & Wenger, 2004) in the sense that the facilitation of the total processing efficiency benefit from interactions between internal sub-processes was so strong that it was even beyond what the stochastic maximum statistic in fact could predict (see Colonius & Vorberg, 1994; Townsend & Nozawa, 1995, for detailed proofs).

Next, we turn to capacity coefficients simultaneously accounting for both response time and accuracy. The $A_{CF}^{AND}(t)$ functions were calculated using the Townsend–Altieri decomposition method (see Eq. 7; also see Supplementary Table 17 for upper and lower bounds in decomposed formats). As demonstrated in Figure 5a and b, $A_{CF}^{AND}(t)$ was also found to be above the prediction of UCIP model in both unbiased and biased conditions. In other words, even after accounting for both response time and accuracy performance, an improvement of the total processing efficiency of cognitive systems in the double-channel condition were evidenced. These results imply a super capacity processing, and again suggest that processing of both height and width can benefit the total processing efficiency in the presence of rectangles. It is intriguing to observe that the values of $A_{CF}^{AND}(t)$s were substantially lower than $C(t)$. We thereby infer that although still super capacity, accuracy suffered when compared with processing time in the presence of increased workload.

Finally, both $C(t)$s and $A_{CF}^{AND}(t)$s tended to be greater in the Biased 2 condition favouring target responses than in the neutral and, more mildly, in the neutral than in the Biased 1 condition favouring contrast-target responses. When people shifted from the unbiased condition to the Biased 2 condition, the response times decreased stochastically for three out of four observers (see Table 27), indicating an improvement of the processing speed. On the other hand, when observers shifted from the unbiased to the Biased 1 condition, half of observers became significantly faster, while the other half became significantly slower.

Conclusion. In sum, the results presented above from the classification task demonstrate that cognitive processes of height and width overwhelmingly followed a parallel-exhaustive stopping rule in the unbiased condition. Moreover, the attendant parallel channels of these observers were endowed with extreme super capacity (Eidels et al., 2011). The classification findings imply that rectangle height and width are not processed in an independent and certainly not in a serial manner. Super capacity parallelism accompanied by an exhaustive stopping rule are associated with information processing interpretations of configurality (Townsend & Wenger, 2015).

General summary and discussion

Our intention in this research was to investigate the cognitive principles of configurality by placing stimuli that can readily be specified in terms of two elemental physical dimensions under our theoretical and methodological lenses. We wanted to employ dimensions that have received some attention in the past from experimentalists and that, though simple, have been deemed to be processed in some

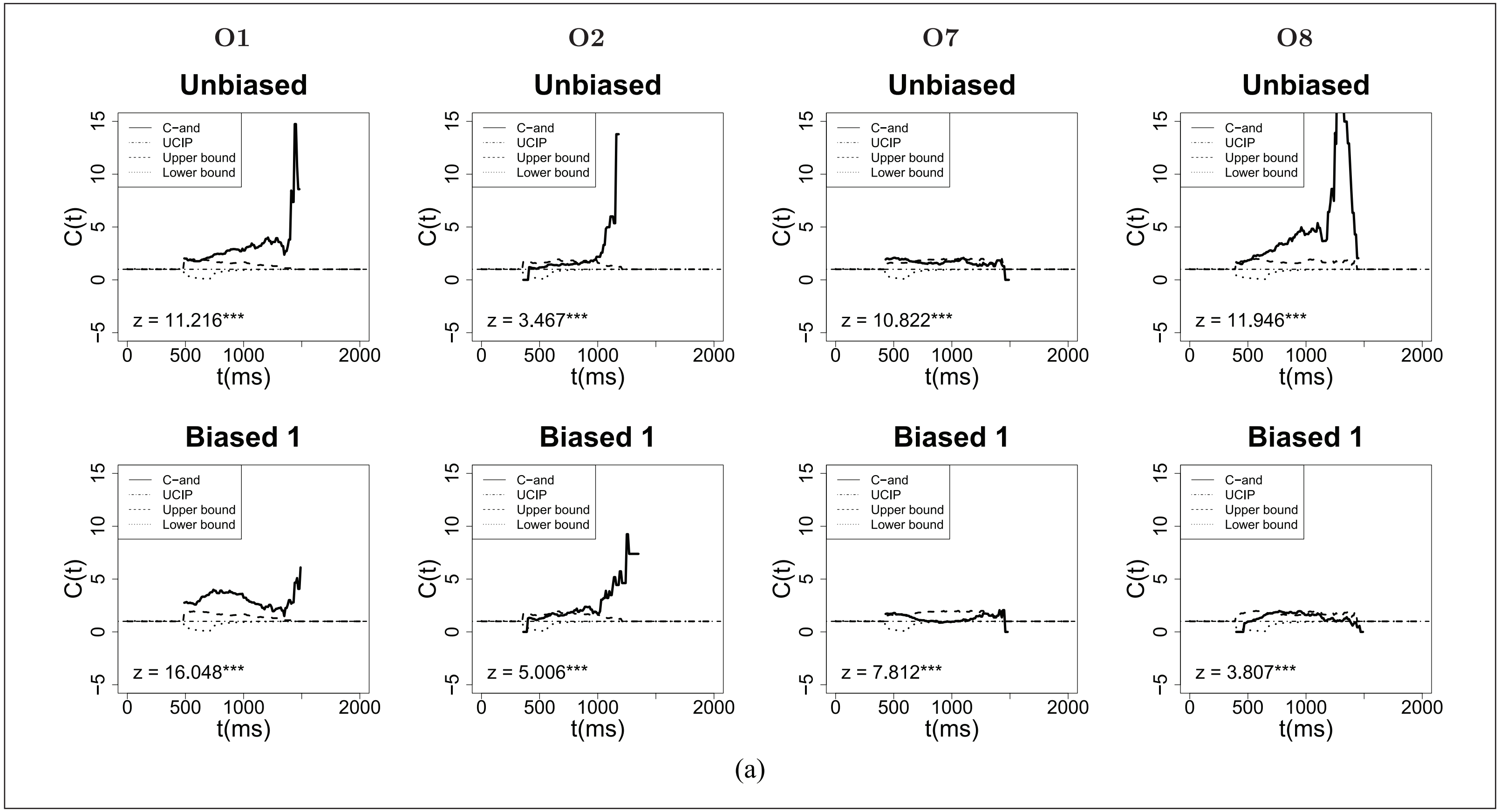

Figure 4. (continued)

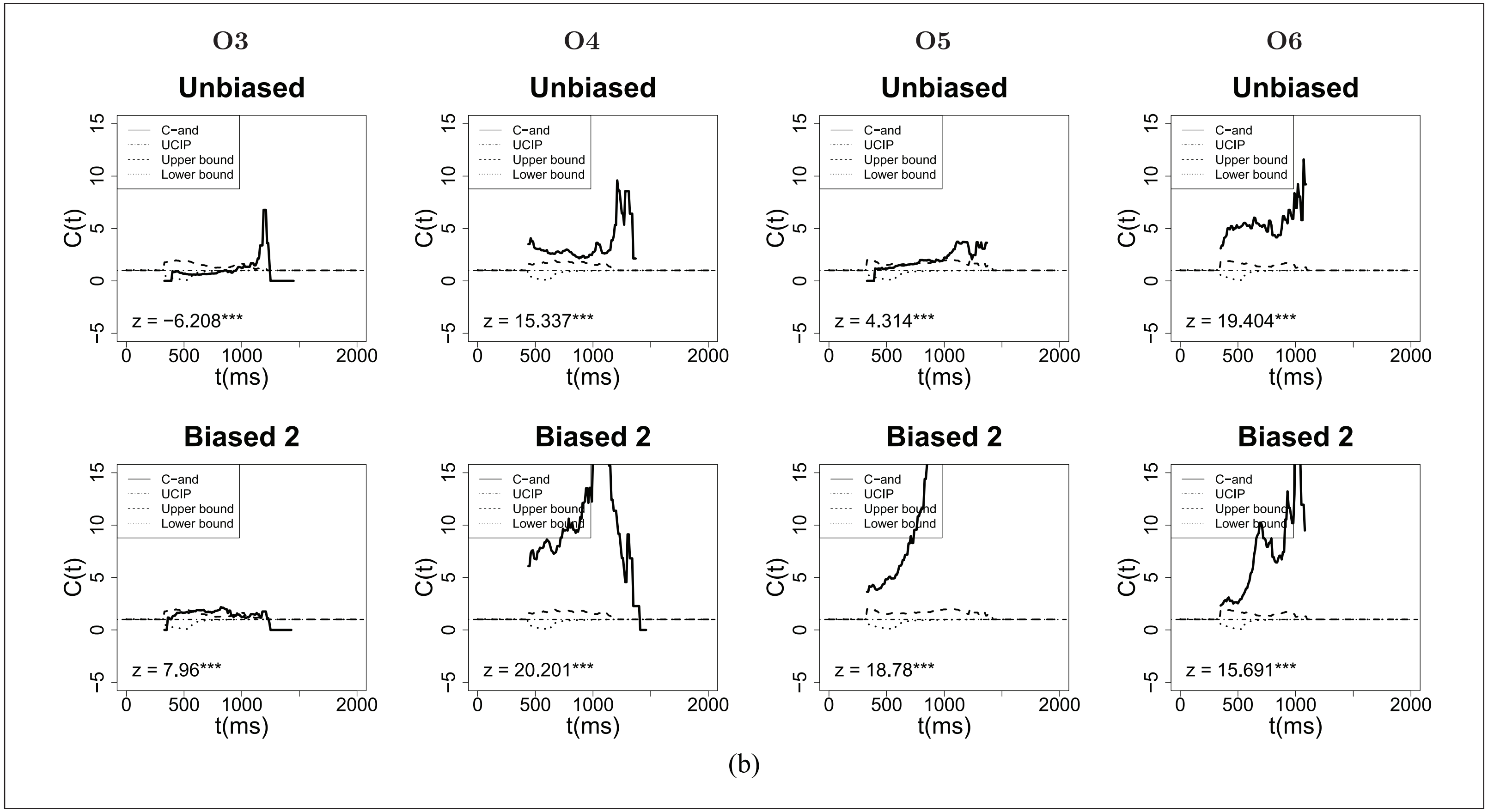

Figure 4. (a) Estimated $C(t)$ s of each observer who completed an unbiased and the Biased 1 condition favouring contrast-target responses. Stars indicate the statistical significance of the deviation of an estimated$C(t)$ from the UCIP predictions. (b) Estimated $C(t)$ s of each observer who completed an unbiased and the Biased 2 condition favouring target responses. Stars indicate the statistical significance of the deviation of an estimated $C(t)$ from the UCIP predictions.
***$p < .001$, **$p < .01$, *$p < .05$.

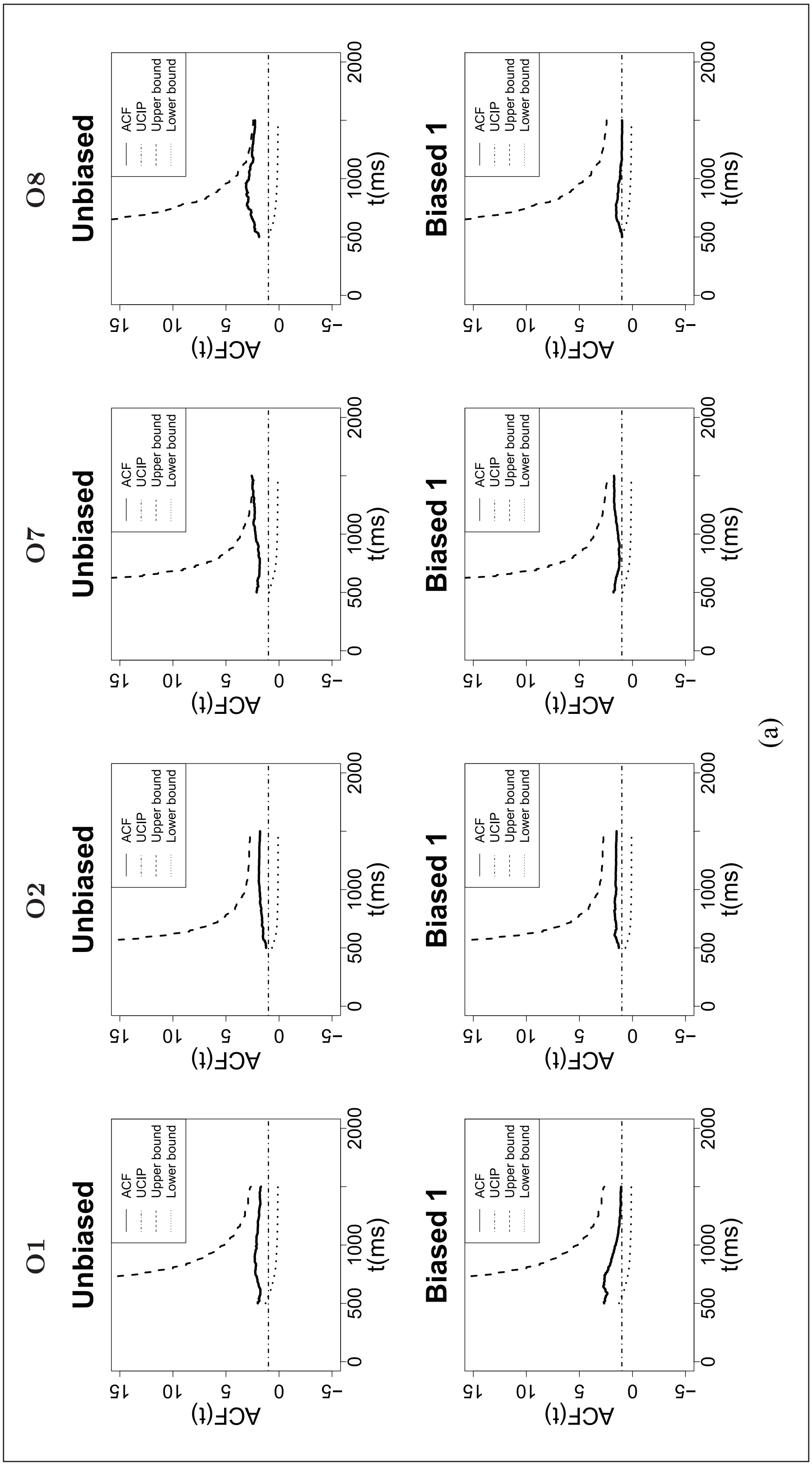

Figure 5. (continued)

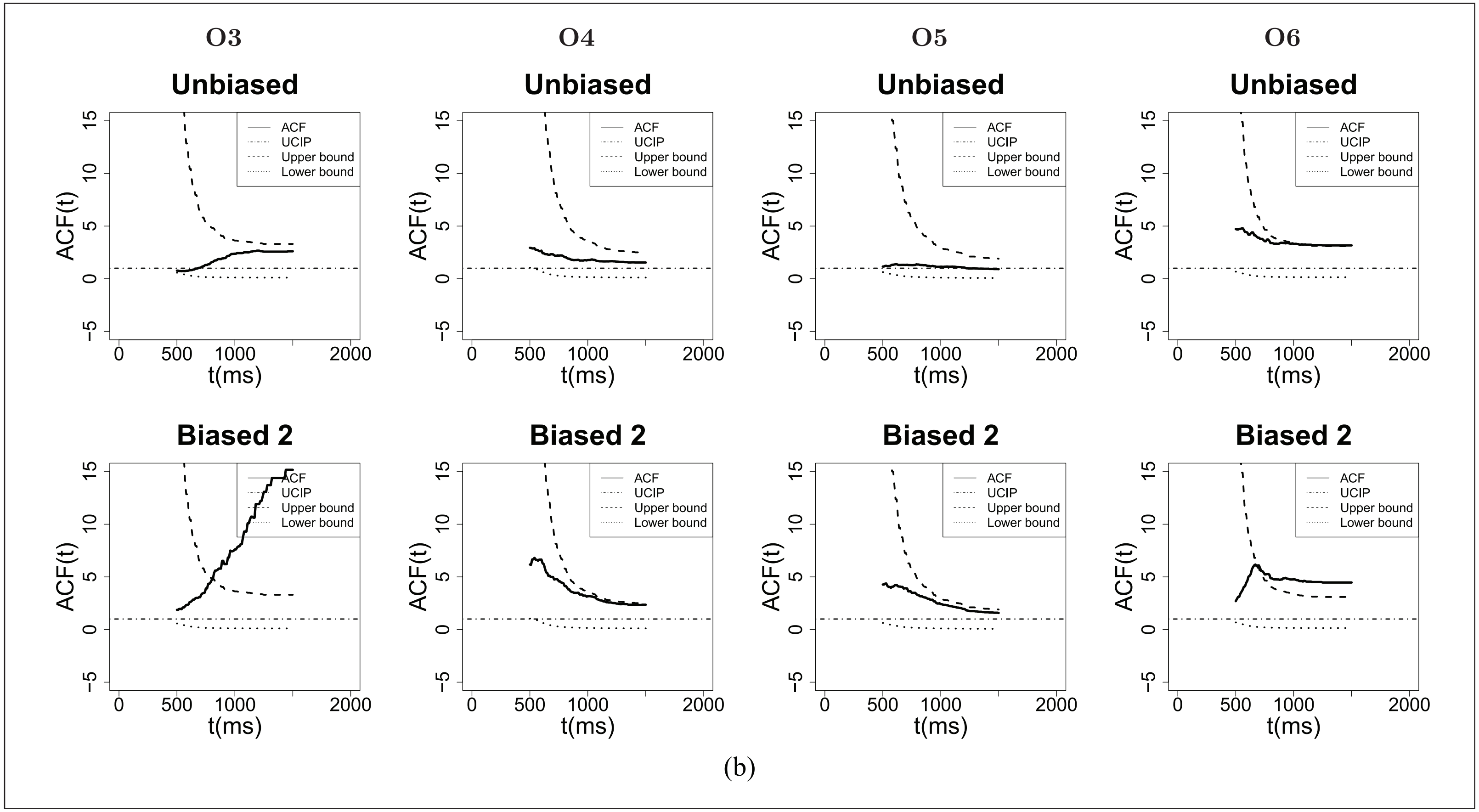

Figure 5. (a) Estimated $A_{CF}^{AND}(t)$ s of each observer who completed an unbiased and the Biased 1 condition favouring contrast-target responses. (b) Estimated $A_{CF}^{AND}(t)$ s of each observer who completed an unbiased and the Biased 2 condition favouring target responses.

Table 27. K-S test results of comparing the distributions of response times given target stimuli between the unbiased and biased conditions.

Obs	Biased Cond.	$\overline{RT}_{Unbiased} - \overline{RT}_{Biased}$	$\hat{D}-$	p-value	$\hat{D}+$	p-value
O1	Biased 1	36.667	0.207	<.001***	0.023	.702
O2	Biased 1	12.141	0.067	.042**	0.008	.957
O3	Biased 2	103.912	0.385	<.001***	0.010	.943
O4	Biased 2	102.503	0.339	<.001***	0.000	1.000
O5	Biased 2	163.479	0.486	<.001***	0.000	1.000
O6	Biased 2	−27.556	0.030	.491	0.242	<.001***
O7	Biased 1	−37.507	0.013	.885	0.107	<.001***
O8	Biased 1	−90.029	0.000	1.000	0.222	<.001***

***$p < .001$, **$p < .01$.

kind of configural manner. Parallelograms and in particular unequal-sided rectangles and squares were our target. Our foremost set of goals aimed at enlisting our theory driven methodologies to attempt to uncover essential properties of the information processing mechanisms involved in the identification and classification of simple figures.

Moreover, we endeavoured to conjoin two separate methodologies which focus on identification of distinct mechanisms, one focusing on dimensional or feature interactions (GRT), and the other, dynamic aspects such as mental architecture (SFT). Aside from two published accounts (Wenger et al., 2021; Wenger & Rhoten, 2020) and one or two projects currently underway of which we are aware, this is the first time these two branches have been united. The underlying characteristics of interest are perceptual separability, perceptual independence, decisional separability, mental architectures, decisional stopping rules, and workload capacities. In addition, we also employed different payoff manipulations to examine the effect of rewards on perception and deliberation.

As our hierarchy of analyses unfolds, indeterminacies that exist at a coarser level are decided by more micro-level testing. We started with the purely nonparametric statistics—marginal response invariance and report independence—including the traditional static versions as well as the newer timed versions. The marginal response frequencies pointed to a dramatic tendency to report stimuli as taller if their width was narrow accompanied by an even stronger tendency to report stimuli as wide if their height was short. Furthermore, observers' results with timed marginal response statistics exhibited stochastically faster responses with the preferred dimensional combinations. The direction of failures of the marginal invariances were such that they could have originated either from violations of perceptual separability or of decisional separability.

Now consider within-trial dependencies, that is, the concept of perceptual independence. The report independence statistics and tests found a striking positive dependence between width and height perception but only for squares, not for rectangles. Indeed, the response times also indicated positive dependencies in ways matching the response probability correlations.

Following these analyses, we applied classical assays of marginal signal detection parameters, the normalised difference of means (d'), and the decisional criteria, expressed as number of standard deviates from a mean of 0 (Kadlec & Townsend, 1992; Townsend et al., 1988). Intriguingly, we discovered that violations of both perceptual separability and of decisional separability contributed to the statistically significant violations of marginal response invariance. However, the sensory influences were seen to be rather more powerful in this type of influence than were the nonetheless discernible failures of decisional separability.

Finally, the individual model fits of complete Gaussian signal detection models cemented the more descriptive inferences, including instances where the d's were greater for width if height were short and for height when width was narrow. And, the placements of the criterion parameters violated decisional separability in a fashion consistent with the earlier analyses, including showing a striking overall preference for rectangles, producing violations of the marginal invariances, and moving moderately in the direction encouraged by the bias conditions. All these findings provide intimate details of critical types of perceptual and decisional integrality.

Before proceeding to our conditions and analyses concerning architecture, capacity and stopping rule, it is important to consider potential issues of model mimicry within the GRT theory. This framework envelopes a very broad spectrum of types of perceptual and decisional interactions and thus it should not be surprising that the spectre of mimicry could arise. R. Thomas and Silbert have investigated a number of such avenues (e.g., Silbert & Thomas, 2013, 2017; Thomas, 1996).

Evidently, the most pertinent to the present results would be their recent theoretical demonstration, accomplished with precise analytic tools that, assuming linear decision bounds, failure of decisional separability combined with perceptual separability can be transformed to a model satisfying decisional separability but failing perceptual separability. Similarly, but somewhat more limited, was their proof that an important kind of breakdown of perceptual separability aligned with decisional separability could be converted to a configuration obeying perceptual separability but violating decisional separability. This type of perceptual separability disruption is known as mean shift integrality. This kind of integrality is produced by transforming the classic rectangle arrangement of distributional means, to the more general parallelogram.

It does not seem that the Silbert and Thomas (2013) framework will readily accommodate our rather complex, but quite orderly and interlocking mesh of findings. As documented above, the decisional separability disturbance was best explained by a set of piecewise linear but orthogonal, decision bounds and this result was accompanied by a loss of perceptual separability not captured by d' s indicative of a parallelogram. In particular, Supplementary Appendix B exhibits tests of mean shift integrality revealing that as a possibility for (only) one participant. Attempts to alter our general conformations through machinations such as translations of the means also led to patterns of perceptual dependencies (e.g., as viewed in terms of response independence statistics) that were no longer easily interpretable.

Now consider an overview of the classification task and associated analyses. Our analyses permit the inference from the data of both the unbiased condition and Biased 1 condition favouring contrast-target responses that observers' *SIC* (*t*)*s* are unambiguously parallel with an exhaustive stopping rule on the dimensions of height and width. However, when observers shifted to the Biased 2 condition objectively favouring target responses, the combination of architecture and stopping rule were more equivocal.

It is puzzling as to how and why a manipulation of the response bias could significantly perturb the architecture and/or stopping rule. C-T. Yang et al. (2014, 2019) explored the change of mental strategies with the validity of attention cues and showed that observers tended to employ a parallel strategy for low valid cues but serial for high valid cues. Their findings reveal an intriguing relationship between the attention controls and mental architectures and may partially account for the current observations. Yet, there is still a lack of explanation for shifts in stopping rules and increases in the processing efficiency. More research will likely be required to decide on this issue.

Processing of height and width were found to facilitate each other in the classification task. As suggested by both capacity measurements $C(t)$ and $A(t)$, an evident improvement of processing efficiency, as measured purely by response times or jointly by accuracy in addition to response times, were observed in the double-dimensional condition in comparison with single-dimensional conditions. Consider our current tactic for measuring efficiency of processing of a single dimension, namely using separate trial-blocks requiring participants to focus on a single dimension. This can be contrasted with the usual double factorial paradigm where the participant is attending to both dimensions but only one is presented on certain trials. It would be expected that, if anything, the present technique could lead to lower capacity assessments. But, we still measured super capacity which makes this finding even more convincing.

In principle, this facilitatory interaction could come about either through the advent of extra and new resources being made available or, much more likely, through positive channel interactions (see Figure 3 of Eidels et al., 2011). A rather extreme form of channel interactions, but one long considered to be consistent with integrality, is coactivation (e.g., Diederich & Colonius, 1991; Miller, 1978). Coactivation occurs when two parallel channels, each of which operates just as efficiently when the other channel is occupied as when not, add their activations into a final, pooled conduit which is compared with a decision threshold (e.g., Houpt & Townsend, 2011; Miller, 1978; Townsend & Nozawa, 1995). The SIC characteristic of coactive systems would exhibit a small negative blip early on before changing to a large positive portion thereafter, which is not at all like the purely parallel (negative SIC) curve found here.

The greater sensitivity of width with short height and height with narrow width are not visible in the SFT data nor is the "preference" for rectangles over squares. Neither datum is expected to be apparent through SFT. However, the positive dependencies found in GRT are definitely harmonious with the super capacity discovered with SFT. Such interpretations pose one of the few well-defined quantitative indicants of holism (Fific et al., 2010; Townsend & Wenger, 2004, 2015; Wenger & Townsend, 2006).

As reviewed in the Introduction, there are few aiming for uncovering multi-dimensional aspects of object perception such as of rectangle perception. There may nonetheless be some regions of overlap. For instance, our conclusions regarding greater bias and sensitivity to a larger judgement when the alternative dimension was smaller may correspond to Piaget's (1969) findings that height would be overestimated if paired with a narrower width than with a wider width.

Much of the earlier research used the MDS or Garnerian methods. One aspect from the scaling and closely related literature that seems to dovetail with our inferences is that a number of studies remark on the greater salience or efficacy of shape vs. size (e.g., Feldman & Richards, 1998; Weintraub, 1971) when the focus is only on those two dimensions.

Macmillan and Ornstein (1998) implemented Garner's operational concepts and used GRT principles (Ashby & Maddox, 1994; Maddox, 1992) to avoid the priori of different mechanisms caused by distinct experimental operations in conventional Garner methods. One hallmark of the Ashby and Maddox GRT strategy is that the multidimensional geometry vis-a-vis signal detection sensitivity and decision parameters predict results for the non-baseline conditions that a strict Garner construal would view as implying integrality. These general precepts were obeyed in the Macmillan and Ornstein (1998) data and demonstrate that GRT can predict what would be dubbed integral from the original Garner perspective, when the GRT characteristics are actually separable.

Nonetheless, the positioning of the GRT distributions did reveal a straightforward type of integrality. Thus, results from Macmillan and Ornstein's (1998) Garnerian conditions were consistent with a rhomboid type of parallelogram (unequal sides and non-right angles), which signals, according to the Ashby and Maddox GRT standpoint, the presence of a mean shift integrality configuration. In this case, the angles were such that the diagonal distance between rectangles (primarily a shape distinction) was considerably greater than the distance between the squares (an area distinction). Of course, we have to keep in mind that their inferences here, may have been contaminated by the untested presence of perceptual dependence. Although we did not find a mean shift integrality layout of our GRT distributions for most observers, a very salient conclusion was the considerably larger distance between rectangles than between squares, just as in their case.

What can we conclude about the primary methodologies heretofore employed outside of GRT and SFT with regard to promise for uncovering multi-dimensional aspects of object perception? With regard to multidimensional scaling of parallelograms, the nature of investigators' inferences over the years have seemed to fluctuate rather markedly. In addition, appraisals by experts of what are acceptable transformations to visit upon humans' judgements of dissimilarity vary radically and can dramatically affect the scaling solutions. Perhaps use of dependent variables on stronger scales, such as relative frequencies or response times might offer a remedy for some of these challenges (Podgorny & Garner, 1979; Townsend, 1971).

The Garnerian methods seem likely to be more efficacious, especially when allied with GRT and SFT approaches. Thus, as noted and utilised above, the perceptual layout of the GRT distributions are quite informative especially when conjoined with response times within RT-GRT as well as SFT. Further, a nice property of the Garner approach is that certain conditions look for configurality through facilitation (his standard correlated condition, and cf. Townsend & Wenger, 2004) whereas others reveal configurality through interference (his filtering condition) and yet others utilise comparisons of classes of patterns (e.g., his divided attention condition). Yet, our theoretical stance views these varying experimental conditions as potentially shedding light on distinct aspects of the information processing apparatus rather than simply testing for apparently converging agreement, in analogy to the Macmillan and Ornstein (1998) analyses.

The issue of somewhat distinct paradigms crops up here as well. Although both our projects employed stimulus patterns that overlapped in identity and we used the same participants, the distinctions between the separate tasks opens questions concerning a fully unified interpretation between our GRT and SFT phases. For instance, the differences in the number of dimensional levels (two in the identification vs. three in the classification) and the response type (i.e., four-alternative responses in the identification vs. two-alternative responses in the classification) could lead to under or over estimations of the response efficiency when comparing the behavioural performance between the two tasks.

It would be beneficial to construct an experimental design which requires complete identification but where the stimulus salience (intensity, etc.) of the dimensions are manipulated to also permit SFT analyses. A recent investigation basically employed a complete identification (GRT-like) design where a participant had to identify the presence or absence of each dimension in each stimulus (Howard et al., 2021). However, that study only focused on the SFT analyses. We are in the process of developing experimental paradigms wherein SFT and GRT are truly unified. By this we mean not just by running the same observers in identification and classification designs using the same stimuli, but where the same trials simultaneously probe the GRT *and* SFT concepts. Hopefully, the future will bring more research including direct comparison of design-results to begin to provide an overall picture of human perceptual and cognitive systems.

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Ethical approval

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Consent to participate

Informed consent to participate was obtained from all the observers who participated in the current study.

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Data accessibility statement

The datum generated during the current study are available on Open Source Framework (https://osf.io/srpva/?viewonly=182027d819214e9f92598d64e62d8cdc).

Supplementary material

The supplementary material is available at qjep.sagepub.com.

Notes

1. Garner is one of the pioneers of the new cognitive psychology of the 1950s and 1960s where concepts from what came to be called *artificial intelligence* and the *information sciences* were applied to human perception, cognition, and action.
2. The pioneers of foundational measurement theory have propounded axiomatic conditions for the existence of certain classes of metrics, especially the so-called power metrics (Suppes et al., 1982). Other more recent axiomatic treatments pertinent to issues of integrality and its opposite, separability, include Dzhafarov (2004).
3. For instance, Silbert and colleagues (2009) utilised base rate manipulations, influencing decision bounds, to afford increased identifiability.
4. The studies for GRT and SFT analyses typically apply a design that collects large number of trials from a small number of observers. In such a design, each participant provides a large number of robust measures, which benefits the investigations of systematic functional relationships (Smith & Little, 2018), such as the cognitive principles concerned in the current studies.
5. Such a design followed an extended version of a feature-complete factorial design that was originally employed by Kadlec and Hicks (1998) to infer perceptual separability from various d'. In this paper, we took a similar approach but also utilised a set of theory-driven measurements to infer various dependencies and other cognitive properties, in addition to perceptual separability. To this end, we mainly focused on the results of the analyses performed on data collected from the first five block types in the main body of this paper.
6. The observable non-parametric conceptions of timed marginal response invariance as well as timed report independence are the logical counterparts to their static counterparts. Although the battery of statistical tools available for application to RTGRT are naturally far less rich than for GRT, some advances have been made beyond the basic non-parametric tests put forth in Townsend et al. (2012) general. Thus, the analyses we conducted on data for testing timed-MRI and timed-RI were based on the Brownian-bridge method (Houpt & Townsend, 2010, also see the Supplementary Appendix B for detailed analyses).
7. The marginal d' difference on a dimension was calculated by comparing the d' of a dimension estimated in the presence of low level of the other dimension with the d' of that dimension estimated in the presence of high level of the other dimension. Specifically, the marginal d' difference of Height = d'(Height | Narrow Width) − d'(Height | Wide Width), and the marginal d' difference of Width = d'(Width | Short Height) − d'(Width | Long Height).
8. Extending from the conventional model hierarchy (see Supplementary Figure 6 in Thomas (2001b)) which assumes decisional separability throughout, the model hierarchy used in the current study allows the violation in the decisional separability in a linear piecewise manner (Macho, 2010).
9. See Definition 1, 2, 3 for detailed implementation of underlying postulates of various independencies in multidimensional signal detection models.

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